120th ASEE Annual Conference & Exposition
FRANKLY, WE DO GIVE A D*MN
June 23-26, 2013

Paper ID #6778

The 3D Estimator: Introducing Middle-School Students to Back of the Envelope Estimation Interactively

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Introduction

There has been recent concern among post-secondary engineering educators that engineering majors arrive from high school underprepared to assess the reasonableness of estimates, instead relying too heavily on the output of calculators [1]. This echoes research in the education community identifying the theme of the reasonableness of estimates as an important and underutilized concept in K-12 education [2]. In response to these concerns, a new assessment tool, the 3D Estimator, was developed to introduce students to multi-step estimation. This skill is a pre-requisite to the kind of multi-step "back of the envelope problems" that engineers often use to restrict or filter a problem's alternatives in the planning and ideation phases of design [3]. Descriptive and correlational statistics were collected and analyzed on the nature of students' multi-step estimates using the 3D Estimator.

This paper is structured as follows. The next section deals with the objectives of two studies that investigated students' use of the 3D Estimator. After that, background concepts from related literature are covered, including operational definitions of key terms. The subsequent two sections deal with Study 1 and Study 2, respectively. Finally, there is a section drawing conclusions and briefly describing future work.

Objectives

This research-to-practice paper reports on two studies designed to address the question of how middle-school students develop multi-step estimates. A new assessment tool, the 3D Estimator, was developed for students to use in estimating the volume of 3D shapes. With this tool, students estimate aspects including length, width, height, and radius for shapes such as prisms, cylinders and spheres. Finally, they submit a calculated estimate for the overall volume of each shape.

The first study was exploratory in nature, addressing questions of how the students would approach the software. Of particular interest was the amount of time students spent analyzing the shapes from different angles. The second study used correlations between students' error rates on different problem-solving components to assess how systematic their final estimates were. Also collected was the number of estimates that students generated in each series of estimates. For example, if a student was attempting to estimate the volume of a 60 cubic-foot shape, a series of estimates might be 100 cubic feet (too high), then 40 (too low), then 50 (still too low). Estimates were labeled as reasonable if they fell within a +/-15% tolerance range. The +/-15% tolerance was established over the course of previous research as a range that would be forgiving enough to allow for a variety of estimation strategies, but tight enough to force students to think carefully.

One of the key features of the 3D Estimator is its use of feedback. For each estimate that a student makes, the interface provides feedback in the form of a horizontal error bar. The bar indicates relative error, extending to the left from a central point for estimates that are low, and extending to the right from the center for estimates that are high. There is a red box around the center of the graph to show the +/-15% tolerance region. Estimates that are deemed reasonable will have an error bar that is within the red box. Once a student makes an estimate that is within this region, the actual exact answer is shown in the display (see Figure 1).

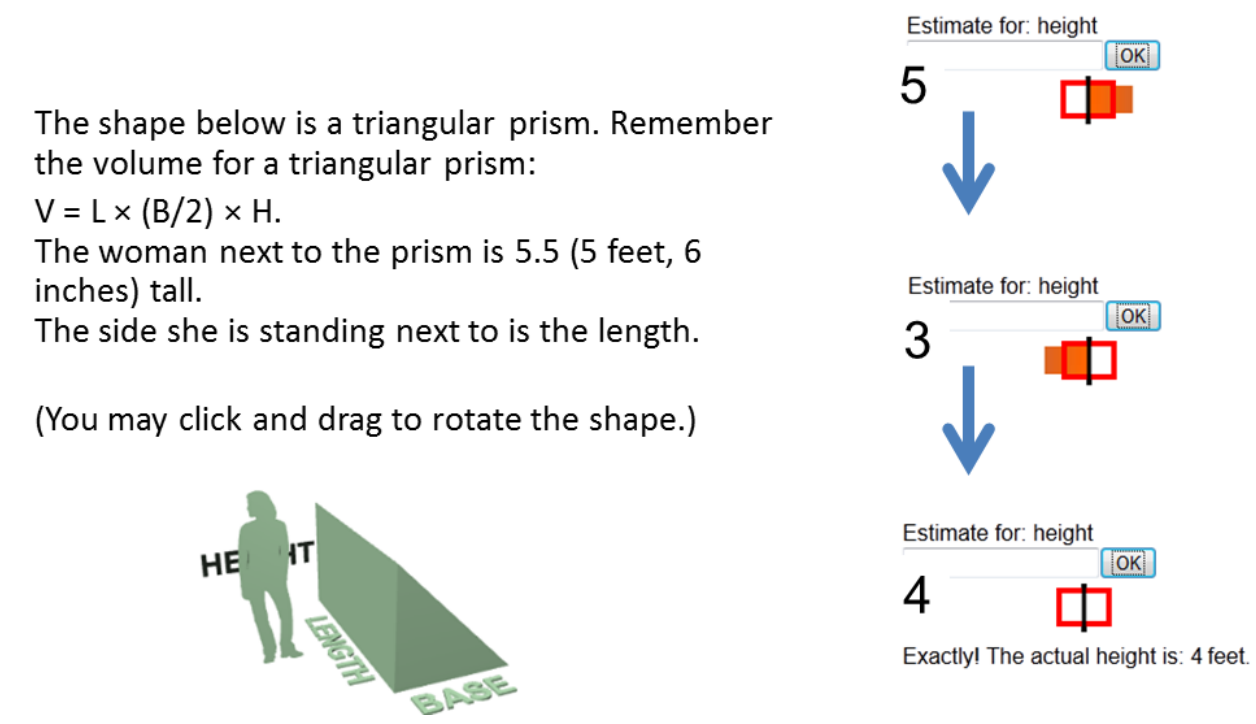

Figure 1. The 3D Estimator. The left half shows the information that is shown to the student, including the manipulable 3D shape, the human figure and labeled dimensions, as well the shape's name and formula. The right half shows a sequence of estimates made by a student for the height of the figure and the orange error feedback bars for both of the initial estimates (overestimate of 5 and underestimate of 3).

Background

Knowledge Components

This description of the present research refers to elemental chunks of known or to-be-learned information as knowledge components, or KCs [4]. KCs can consist of simple factual knowledge such as: "the volume of a rectangular prism is the product of its length, width, and height". KCs can also be production rules, which are if-then statements that are used to perform actions, such as: "if the light is red, stop the car." Anderson[5] points out the functional equivalency of factual KCs, which cognitive psychologists call declarative knowledge – and production rule KCs – or procedural knowledge. A factual KC can be stated as a production rule KC, such as: "if you are asked to find the volume of a rectangular prism, first establish its length, width and height, then calculate the product of all three." However, it is more efficient to encode the volume of a rectangle as a factual KC, such as length × width × height, rather than the more elaborate rule starting with "If you are asked to find…"

The 3D Estimator is concerned with assessing the production rules associated with "if you are asked to *estimate*…" which has a larger space of possible valid approaches than "asked to *find*". Finding or calculating an exact answer usually elicits one of a small set of standard arithmetic algorithms from students, or the use of a calculator. On the other hand, if students are asked to estimate an answer, students may choose from a larger set of strategies [6, 7]. The present research is more concerned with students' estimation strategies than with memorization of factual KCs such as the formula for the volume of a rectangular prism. For this reason, the 3D estimator provides such formulas in the interface (see Figure 1).

Estimation & Fermi Problems

Fermi problems, or "back-of-the-envelope" problems are named after the physicist Enrico Fermi (1901 – 1954). They are quick, mathematical approximations to real world problems that depend on a sequence of estimates, combined via computation [8]. Educationally, they are often used in engineering classes because they are thought provoking and because practicing engineers often make similar approximations as a way of restricting or filtering a problem's design space [3]. In practice, a quick approximation can often provide insights into the feasibility of a design approach.

Calculations used in solving Fermi problems tend to be based on estimates and answers depend on computational estimation. A classic example of a Fermi problem is *how many gallons of gasoline do all motor vehicles in the U.S. use in one year?* The answer can be calculated in a variety of ways, using known or estimable information, such as [(U.S. population) × (vehicles per capita) + (number of U.S. private or public establishments) × (vehicles per establishment)] × (mean miles driven per day per vehicle) × (mean fraction of a gallon per mile). This is a relatively simple expression for solving this gasoline Fermi problem. Solutions with more terms have the potential to be more accurate, but may introduce extra complexity and can eventually violate the spirit of Fermi problems. By convention, a Fermi problem should be able to be calculated on the back of an envelope – that is, no more than a handful of calculations should be involved.

By their nature, Fermi problems depend on the use of some prior knowledge. Students must be able to perform the following steps: 1) conjure up relevant values such as the approximate U.S. population or MPG of a car, 2) understand the necessary mathematical operations to perform on these values, 3) use those operations in a logical and cohesive mathematical way, and 4) reflect on whether the estimate might or might not be reasonable. The kinds of problems presented by the 3D Estimator primarily assess students' performance of the third step, thereby assisting with performance on the fourth. That is, the 3D Estimator assesses students' use of mathematical operations and numerical strategies for producing reasonable estimates. Producing reasonable estimates requires a flexible understanding of the number system, and relies heavily on number sense and skills such as the ability to work with powers of 10 [9]. These fundamental estimation skills are first taught in primary grades, but they are often found to be lacking in adolescents and teenagers [10].

There has been recent concern among some post-secondary engineering educators that engineering majors arrive from high school underprepared to assess the reasonableness of estimates, instead relying too heavily on the output of calculators [1]. This echoes a study surveying 40 U.S. teachers who see the theme of the reasonableness of estimates as an important and underutilized concept in K-12 education [2].

Reference Points

An understanding of the reasonableness of estimates is an invaluable metacognitive tool when a student is attempting to generate his or her own original estimates. Such understanding is referred to in the present work as a knowledge component (KC) and this is certainly a multifaceted one, referred to in related literature as an integrative KC [11], meaning one that is composed of smaller KCs. In order to assess the reasonableness of an estimate, it helps to have an understanding of related values. For example, for a student to estimate the dimensions of his or her gas tank, it may be helpful for that student to think of related volume measures that might be more familiar. If a student can picture twelve gallon-size bottles of milk, or a half-filled 20 gallon aquarium, he or she may have an easier time estimating the dimensions of a 12 gallon gas tank. This kind of familiar point of comparison is sometimes called a reference point [12] or, more broadly, an anchor for estimation [13].

An iterative check on reasonableness increases the robustness of estimation. That is, a student may use one strategy to generate an estimate, and then use another strategy to check its reasonableness. A student who estimates that a 12 gallon fuel tank is 3 feet × 3 feet × 3 feet will recognize that these dimensions are not reasonable when he considers that a fuel tank must be small enough to fit underneath a car.

The 3D Estimator was designed as an assessment instrument. However, one of the questions addressed in the present research is whether noticeable learning occurs between problems. In other words, do students improve from one problem to the next? Since each problem presented in the 3D Estimator concerns the use of volume estimation, we hypothesized that students might be able to use the answer from previous problems as reference points. As described below, Learning Factor Analysis [4] was used to investigate this hypothesis.

Floundering

One of Anderson, et al.'s [14] definitions of floundering with interactive learning software is repeating the same kind of mistake three times (p. 174). This definition proved useful for analyzing the student results with the 3D Estimator. The 3D Estimator follows in a line of research initiated with the Estimation Calculator [15]. Both depend upon providing feedback on the reasonableness of an estimate, as measured by its relative error, $\frac{E-S}{S}$, where S is the exact solution, and E is the estimate for the solution. Log files store student activity data, including the series of student estimate errors, which will, if students are paying close attention, eventually fall within +/-15%. Students were shown the relative error of each estimate in turn, and used a combination of this error feedback and their understanding of the problem to produce a revised estimate. When an estimate converges on +/-15%, the 3D Estimator provides the exact answer, so a student stops making estimates at that point.

The +/-15% tolerance was established over the course of the research as a range that would be forgiving enough to allow for a variety of estimation strategies, but tight enough to force students to think carefully. It is more restrictive than traditional Fermi problems, which are often evaluated based on order-of-magnitude reasonableness [3]. However, for the small-magnitude values of the problems under investigation here, this tolerance is more informative and useful for students. Research with the Estimation Calculator indicated that students with a tenuous grasp of number sense would make large numbers of estimates, sometimes as many as seven estimates, relying heavily upon the feedback in the display rather than carefully rethinking estimates whose error was large. The term floundering is used to describe entering more than three estimates that do not converge on the +/-15% tolerance range.

Computational and Measurement Estimation

Making estimates for aspects of the shapes and making estimates for the volumes suggested two distinct abilities. Measurement estimation and computational estimation have been found to be independent abilities [16]. Estimating aspects, such as length or radius, is a form of measurement estimation [12]. Estimating volume can be a form of computational estimation [17] because it requires the synthesis of the previous aspect estimates combined via geometric formulas. For example, a student might perform measurement estimation on a rectangular prism with dimensions of approximately 3, 4 and 5 feet. Then she might perform computational estimation to estimate 50 cubic feet as the volume of the 3 foot × 4 foot × 5 foot prism. She might first reformulate 3 × 4 × 5 to 10 × 5 because 10 is close to 12.

Spatial Ability

Mental rotation ability is a subset of spatial visualization skills [18]. It predicts engineering abilities and has been found to be mutable and teachable [19] [20]. It is defined as the ability to mentally rotate shapes and to imagine them in different orientations. In order to form estimates with the 3D Estimator, a student must translate a measure shown in one dimension to other dimensions (see Figure 1). The height of each shape in the 3D Estimator can be estimated by comparison with the height of the human figure shown in the display. The shape can be clicked and dragged to different orientations. Meanwhile, the human figure retains its orientation relative to the shape, so that a user looking at the bottom of the sphere will also see the bottom of the feet of the human figure, which is provided. Thus, in order to estimate the dimensions of width and height, a student using the 3D Estimator must rotate the one of the reference points mentally. For example mental rotation of the human figure or another side of the shape would facilitate this estimation.

Ability as Enhancer Hypothesis

Huk [21] and Mayer and Sims [22] attribute the better performance of high mental rotation ability individuals to an ability-as-enhancer hypothesis that holds that high mental rotation ability enhances learning using visualizations. The opposing view is the ability-as-compensator hypothesis that individuals of low mental rotation ability will depend more heavily upon visualizations and thus benefit more from them. Their proposed mechanism is that individuals of high mental rotation ability have more cognitive resources available to form referential connections when working with visualization than do low mental rotation ability individuals. Following the ability-as-enhancer hypothesis, we hypothesized that mental rotation ability would be a moderating variable between the use of a 3D visualization and measurement estimation, and that high mental rotation individuals would spend more time using the 3D geometry visualizations in order to form measurement estimates.

The Use of Log Files for Assessment

The National Education Technology Plan [23] recommends the design of systems that "collect evidence of [students'] knowledge and problem-solving abilities as they work" (p. xi). The plan suggests manipulation of simulation parameters (p. 27) as a use of technology and encourages that systems be created to "learn" about student abilities and readiness. In practice, data can either be stored in memory temporarily for per-session response to user activity, or written to disk for later use. It is the permanent storage of user data in the form of log files stored by server software that we will address here. A study of multi-step estimation problem solving lends itself to extended research study using log files.

There are implicit tradeoffs of quality versus quantity that emerge when a researcher chooses the granularity of a study. The advent of digital technologies allows for extremely fine-grained microgenetic [24] analysis, which seeks to capture data at various points during the period that students attain new knowledge and improve their abilities.

The 3D Estimator

Students use the 3D Estimator in the manner described above, making estimates for each aspect of a three dimensional shape and then making an overall estimate for the volume of the shape. Students are not expected to know the formulas for the volume of each shape. These formulas are provided in the interface and students use them to make estimates for the volume. A sequence of estimates is shown along the right hand side of Figure 1. The feedback in the form of error bars is shown under each estimate, students are shown the exact answer if their estimate is within the tolerance region. As with the Estimation Calculator, the tolerance region was chosen to be +/– 15%.

WISE and WISEngineering

The learning management platforms underlying the 3D Estimator are WISE (http://wise.berkeley.edu) and WISEngineering (http://wisengineering.org). The WISE learning management system allows students to work through a curriculum step-by-step. Steps are individual web pages based on WISE templates, or step-types. This unit included one step-type that was specifically designed for WISEngineering, a new system based on WISE. WISEngineering aims to incorporate knowledge integration [25] with engineering pedagogy [26]. Mathematics and science units developed for WISEngineering scaffold students' engineering projects in a systematic and iterative fashion. WISE has been built to support visualizations for teaching science [27]. A natural extension for WISEngineering was the incorporation of visualizations for mathematics. The 3D Estimator uses these new mathematics visualization capabilities to allow students to practice estimation.

Study 1

Research Question

RQ1) How do students describe their own strategies when solving multi-step geometry estimates?

RQ2) Do interactive 3D *visualizations* differentially support students of high and low mental rotation ability in solving multi-step 3D geometry problems?

Participants

Participants in this study included 34 eighth graders at a private K-8th school. This was a convenience sample; the researcher knew the earth science teacher of these students and administered the online activity during time usually designated for earth science lessons. The students belonged to two different sections of the eighth grade class. In this study the two sections were treated as distinct groups – the calculator group and the no-calculator group. This split was made for purposes of convenience, although undetected systematic differences may exist between the two groups.

Methodology

Activity

The two groups performed the activity on desktop computers running Microsoft Windows, with one student per computer. First, participants were given the Purdue Spatial Visualization Test – Rotations (PSVT-R)[28] to measure mental rotation ability. Six questions chosen at random from the rotations subtest were administered as part of the online activity.

Immediately following this the six 3D geometry estimation problems were presented. The first shape was a cube. The remaining shapes included two other rectangular prisms, a triangular prism, a cylinder and a sphere. Next to each shape was a representation of a person (See Figure 1). The person's height was given to the students in the explanatory text, so that this could be used as a reference point [12]. The shapes could be rotated in all three dimensions by clicking and dragging. The position and relative orientation of the person stayed the same as the student rotated the shape. For example, if the student rotated the shape to be upside-down, then the person would also rotate to be shown upside down. The shapes were represented in perspective. The user's ability to rotate the shape to orient it perpendicularly to the view was expected to reduce foreshortening effects [29] that can arise with similar estimation tasks in virtual environments. Participants in the first group were allowed to use calculators, and several students chose to use the Calculator Windows application. Participants in the second group were prohibited from using any form of calculators and both groups were prohibited from writing anything on paper.

Unfortunately, during this pilot study, the color scheme of the shapes did not render correctly on the PCs. The result was the 3D shapes were displayed as white shapes on a white background, instead of the intended green-on-white. This meant that shape surfaces that were orthogonal to the field of view were invisible, while outlines and sides the user could see at an oblique angle were visible as a shade of gray. Needless to say, this made the task of estimating the dimensions of the shapes in the exercises more difficult than they were intended to be.

Analysis and Results

Students' written descriptions of their strategies were captured in log files. These descriptions varied in detail. Some students wrote one-word answers, such as "guess". Others provided more nuanced explanations, such as the following: "i found the area of one side and multiplied by the height but i did the math in my head sop *[sic]* it wasn't correct." In general, explanations were eloquent and expressed an understanding of key geometry concepts such as congruency and proportion, such as: "I guessed the value of the length, and figured out the width and the height from eyeballing what looked proportionally correct to me." Finally, there were purely mathematical responses, such as: "11x11x11=1331".

Spatial visualization ability was compared between the two groups. The calculator (n=16) and non-calculator (n=18) groups did not differ significantly in mental rotation ability, $F(1, 32) = 1.05$, $p = .31$. Nor did they differ in time spent rotating the 3D shapes on screen, $F(1, 32) = 1.34$, $p = .26$. Finally, using Fisher's r-to-z transformation, a comparison of the correlations within each group of mental rotation ability and time spent rotating shapes was non-significant, $z_{difference} = .10$, $p = .92$. Thus data from the two groups was aggregated to answer research question RQ2.

Mental rotation ability was significantly correlated with total time spent rotating the 3D geometric shapes on the screen, $r(34) = .35$, $p < .05$. This matches results from [21] indicating that students with high spatial ability will use 3D models for longer periods of time than students with low spatial ability. This lends further support to the ability-as-enhancer hypothesis [22].

Discussion

The variety of student answers highlighted the need to allow students to explain their answers in multiple ways. Estimation of this sort involves the synthesis of geometric concepts such as congruency and proportion, which students mentioned. It also involves the straight mathematical computations, such as multiplying 11×11×11, as one student explicitly wrote. Subsequent research will provide students with space to describe their work using both words and formulas separately. It is hoped that students will refer explicitly to reference points that they use, as well as mathematical strategies, if they are prompted appropriately in the software.

The fact that students of higher mental rotation ability spent more time physically rotating the shapes on the screen may have been in part an artifact of the difficulty inherent in seeing those shapes. The ability-as-enhancer hypothesis suggests that students with high mental rotation ability will use visual aids such as these 3D shapes in more effective ways than students with lower mental rotation abilities. Perhaps students with greater mental rotation ability found the tasks more inherently interesting and were therefore more willing to devote more time to "playing" with the 3D shapes on screen.

There is value in seeking viable scaffolds and supports for those students with limited mental rotation ability so that they can find success in science, technology, engineering and mathematics. This research lays the groundwork for online systems that assess students' spatial ability first and then tailors instruction to the needs of the learner. If students are found to have low mental rotation ability, software may adjust activities dynamically so that problems are solved via symbolic rather than visuo-spatial means. Since mental rotation ability is mutable and research has demonstrated methods for its enhancement, [20] curriculum could also be altered to respond to formative assessment.

Study 2

The second study relied on descriptive statistics, as well as the use of Learning Factors Analysis (LFA). The inclusion of LFA was predicated on the idea that even assessment instruments can sometimes provide learning opportunities. LFA uses a mathematical model based on logistic regression to tease apart learning on individual items. Learning is operationalized as a reduction in student error across all students for each assessment item corresponding to a knowledge component (KC). For example, estimating the height of a prism might be considered to be a KC, so that each time any student is asked to make an estimate for a prism's height, his or her response is tied to that KC. The technique of LFA is used to confirm whether, for example, estimating the height of a prism is a truly elemental KC, or whether it needs to be divided into smaller KCs, such as estimating the height of triangular and rectangular prisms.

Although the sample size in Study 2 was too small ($N = 59$) to draw conclusions about learning of specific KCs, the LFA technique was used to determine whether certain problem types resulted in similar levels of error across students. It was hypothesized that dimension estimates such as length, width and height would "hang together" as a measurement estimate KC, and would be distinct from combined estimates for volume, which would be a separate volume calculation KC.

Research Questions

The research was designed to address three questions.

RQ1) Do measurement estimations of one-dimensional aspects and computational estimations of three-dimensional volume represent distinct, separable knowledge components (KCs)? For this, LFA was used.

RQ2) What are the relationships between number of aspect estimates, relative error of aspect estimates, number of volume estimates, and relative error of volume estimates? These relationships were intended to shed light on how strategic or haphazard students' estimation behaviors were. A strong correlation between number of estimates and error of estimates would indicate a haphazard approach.

RQ3) How do students use feedback on the errors of their estimates (hereafter estimate-error-feedback, or EEF) for a volume estimation task? Problem solving involved making measurement estimates using a reference point [12] and then synthesizing these estimates through computation estimation. The EEF as error bars were provided for each estimate that a student made (See Figure 1). This included estimates for aspects as well as estimates for total volume.

Methodology

Activity

The research design was exploratory in nature. The intention of the study was to collect descriptive statistics on students' use of the 3D Estimator, and data for correlation and Learning Factor Analysis. All students were given a forty-five minute regular class period in which to complete the activities. The two eighth grade mathematics teachers devoted their class time to the activities for one day, over the course of three consecutive sections.

Participants

Participants included three groups of eighth grade students (*N* = 59) at a central Virginia public middle school. These groups constituted the entire eighth grade, including all levels of mathematics proficiency at the school.

Assessment

Before making estimates with the 3D Estimator, students took the rotations component of the Purdue Spatial Visualization Test. After completing the mental rotations test, students used the 3D Estimator to estimate the volume of six shapes, as in Study 1. In this study, each estimate that a student entered was recorded and stored in the database.

Analysis and Results

The first research question was: Do measurement estimations of one-dimensional aspects and computational estimations of three-dimensional volume represent distinct, separable knowledge components (KCs)? Determining distinct KCs for the 3D Estimator task requires the use of a learning factors analysis (LFA) and the iterative process of determining q-matrices described by [4]. The analysis shows whether a smooth learning curve exists for a given KC. Smooth curves mean that the entire set of participants makes fewer errors with each attempt. Data were analyzed using DataShop (https://pslcdatashop.web.cmu.edu/). Learning curves were established using only the first estimate in each series. Results were inconclusive. They failed to show a reduction in errors from problem to problem, revealing improvement within each problem instead. Improvement within a problem was to be expected, since making estimates for width (for example) was easier after having converged on a reasonable estimate for height using the error feedback in the bar.

The second research question was: What are the relationships between number of aspect estimates, relative error of aspect estimates, number of volume estimates, and relative error of volume estimates? Correlations were calculated between these four variables (Table 2). These correlations are intended to shed light on the questions of how systematic students have been in the process of making their estimates. They were calculated per student, and relative error of estimates is calculated as mean absolute deviation [30].

If number of estimates and the mean deviation (error) in those estimates is highly correlated, that would suggest a haphazard estimation strategy, or floundering. It implies that the length of estimate series and the error in those estimate series are linearly related. Students who start with very unreasonable estimates require many attempts to converge on a reasonable estimate. Students who start with moderately unreasonable estimates require a moderate number of attempts, etc. There is a significant correlation ($r = 0.45$, $p < 0.01$) between number of volume estimates and error contained in those volume estimates, suggesting that students were haphazard in their approach to volume estimation. However, there is no such correlation between the number of and error in aspect estimates ($r = 0.08$). This does not demonstrate conclusively that students were systematic in estimating aspects; however they may have been able to use cues to converge on reasonable estimates more rapidly than by simply using the error feedback bar.

The other significant correlation that appeared was between the number of aspect estimates and the number of volume estimates ($r = 0.50$, $p < 0.01$). Thus students who made many attempts for aspects of the shapes also made many attempts at estimating volumes. This runs counter to the hypothesis that these two types of estimation represent the independent abilities [16] of measurement and computational estimation.

The third research question was: How do students use estimate error feedback (EEF) for a volume estimation task? An analysis of the distribution of students' estimates was used to address this question (see Figure 2). Estimates were grouped by series, where a series of estimates meant all sequential estimates made for the same aspect or volume. Out of 929 series of estimates for both aspects and volume, 100 did not result in a reasonable estimate. In other words, students stopped or gave up before reaching the +/– 15% tolerance and seeing the correct answer in the output. Patterns of estimation varied widely. On average, students made 2.01 estimates ($SD = 1.67$). However, the distribution of the estimates could be fit very well by an exponential decay curve ($r^2 = 0.96$). The four longest estimate series had lengths of 11, 12, 14 and 17, respectively. More than half (524 series, 56%) contained only one estimate. Of 829 converging estimates, 713 (86%) converged in ≤ 3.

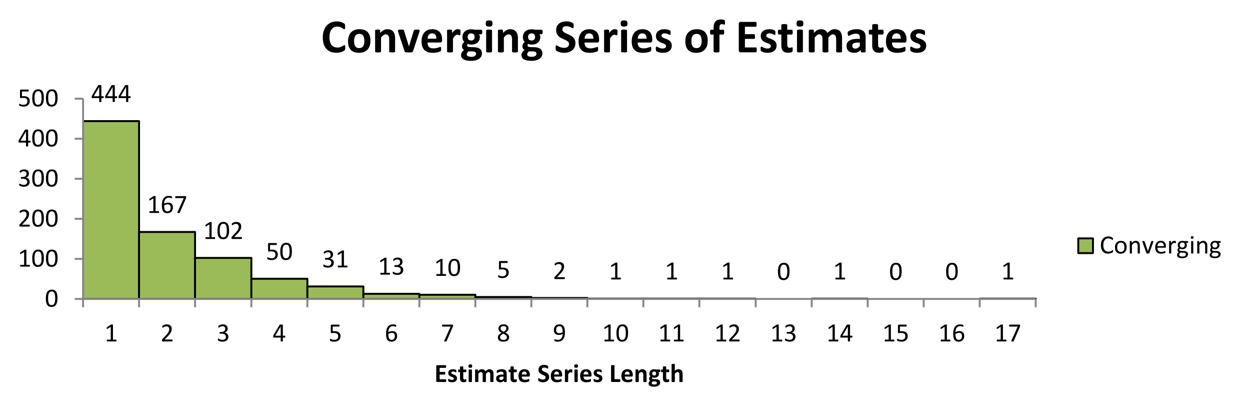

Figure 2: A histogram of students' series of estimates. Converging series reached a final estimate that was within +/–15% of the exact answer. Of these, 86% of estimates converged in within 3 attempts.

Discussion

Overall, the results highlight the need to interrupt floundering [14] among certain students in order to encourage them to think carefully about their estimates. Most estimate series were fruitful. Of the 829 estimate series that resulted in reasonable estimates, 713 (86%) converged on the reasonable tolerance within three attempts. At the opposite extreme, students made as many as 14 or 17 estimates before providing a reasonable one. The correlational analysis suggests that the students who tended to make these long series of unsuccessful estimates made them in similar proportion for shape aspects and for computed shape volumes. This quantitatively supports the notion of floundering established in Studies 1 and 2. Another result, the indication that students are haphazard in their formation of volume estimates also warrants further attention. Finally, the results of the LFA suggest that much care and attention to instructional design, as well as a larger sample size would be necessary to empirically establish KCs and corresponding problems.

Conclusions

In study 1, students with high mental rotation ability tended spend a more extended period of time exploring 3D models than did students with low spatial ability, echoing Huk [21]. These results are possibly confounded by the difficulty that students had in seeing those shapes, which was a result of a technical issue. If this is the case, then the results apply more to the persistence of students of high mental rotation ability in the face of such difficulties.

In study 2, the correlational statistics indicate that students were unsystematic in their approaches to estimation. However, the lack of a systematic approach was not universal. The erratic estimates were largely confined to a few students who made numerous repeated estimates. Those students showed a notable persistence, generating as many as 17 estimates before providing one that was reasonable, as defined by the predetermined tolerance of +/-15%. Such a high number of repeated trials suggests a combined lack of number sense and an inability to generate valid initial estimates, and qualifies as floundering.

The encouraging result in all of this is that students usually do reach reasonable estimates within 3 attempts. A large majority of estimate series (86%) managed to converge on a value within the tolerance range in three estimates or less. This suggests that the technique of providing feedback may be unnecessary until students have had a chance to adjust estimates according to the instant feedback in the display.

Limitations

Small sample sizes limited the generalizability of the results in both studies. Technical difficulties made the shapes in the 3D Estimator more difficult to see in Study 1, which may have contributed to the observed correlation between mental rotation ability and time spent rotating the shapes. The possibility of making estimates based solely on the feedback from the 3D Estimator may itself be a limitation, although one which this research has provided some evidence to help remedy.

Future Work: Prompting Reflection

The next version of the 3D Estimator is currently in development. This version of the software will actually stop students after 3 attempts and encourage them to describe their thought processes. It is believed that this will foster a more metacognitive approach in students. If students make 3 unreasonable estimates, the software will provide text boxes for explaining, both numerically and verbally, what they are doing. This version of the software will also have two different operating modes. One mode will be used for assessment, as described in the research here. The other mode will be used for students to practice estimation, and thereby, perhaps, get better at providing reasonable estimates for this kind of multi-step geometry problem.

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