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Soup Can Races: Teaching Rotational Dynamics Energy-based Solutions

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Abstract

Given our college's urban student population, our students have little exposure to mechanical systems before they take engineering mechanics courses. Classroom demonstrations have been found to strongly support student learning and retention of conceptual ideas.

The following paper outlines a course lecture based on a demonstration titled "Soup Can Races" in which students are asked to predict the order of finish of various soup cans that are raced or rolled down an inclined plane. To aid their decision making, students are reminded of the basic energy equation for the given system stating that the change in potential energy is equal to the sum of the translational kinetic energy and the rotational kinetic energy as well as the moment of inertia for a solid cylinder and hoop about the axis of symmetry.

The soup cans the students are asked to rank, including their masses, are as follows: Crème of Concrete (835 g), Empty Can (51 g), Shell of Can which does not include the can ends (39 g), Tomato Soup (354 g), and Chicken Broth (347 g).

The cans are then raced in front of a loud and very engaged audience followed by a discussion of the results. Typically, the discussion begins with the last place can, Shell of Can, which can be very closely modeled as a hoop. The velocity of the can as a function of the change in height is derived using an energy-based solution. The result is shown to be independent of total mass of the hoop. This is demonstrated by racing the Shell of Can versus a larger and more massive hoop which results in a tie.

Secondly, the Crème of Concrete can is analyzed and compared to a solid cylinder using a similar energy based analysis. The final velocity is again shown to be independent of total mass through the derivation of the velocity of the can and demonstration in class. The velocity of a rolling object released from rest on an inclined plane is presented to only be a function of the geometric mass distribution of the object about the axis of rotation and independent of the total mass.

The Empty Can and Tomato Soup are then analyzed as more complex systems where the rotational energy can be broken into the components. In the case of the Empty Can, the end of the cans and the shell of the can have different rotational energy contributions to the energy balance because the moment of inertia of the solid cylinder or thin disk and hoop are different.

Finally, the remaining question, "Why is Chicken Broth the runaway winner?" can be answered based on the previous discussion assuming that in an ideal case the bulk of the liquid in the can does not rotate.

Introduction

Given the location of our college outside a large urban center, our student population is one of the most diverse in our state. Over 70% of the engineering students who enrolled in engineering mechanics for fall quarter 2012 spoke more than one language fluently. Many from this group are immigrants and first generation college students. As a collective group, they have had very little exposure to mechanical systems. This limited exposure allows for conceptual gaps in their understanding of mechanical systems. To address these conceptual gaps, we implement two Interactive Engagement strategies. These strategies "promote conceptual understanding through interactive engagement of students in heads-on (always) and hands-on (usually) activities which yield immediate feedback through discussion." [1] The positive impact of student engagement on student learning is widely covered in research literature. [1,2,3]

The first strategy is to connect engineering mechanics concepts to occurrences in their everyday lives. This allows us to build on our students' current knowledge. For example, when introducing Newton's second law of motion, we discuss the law of shopping at Costco where most students have been enlisted to push the cart for their families. We draw the connection between the force required to change the velocity of the cart and the quantity of bottled liquid in the cart. Often we also present the same demonstration in class with a chair on wheels. The chair alone can be accelerated with a small force, where as a chair loaded with a student requires considerable force to accelerate.

Secondly, we introduce scenarios with which the students have less familiarity and ask them to make predictions of the behavior of a mechanical system based on their current knowledge of engineering mechanics. We have found that it is important to have students actively record their predictions and verbalize their reasoning behind their choices. This can be done with a show of hands and discussion or more effectively with a written response. Based on the students' documented responses we can begin to help them identify and bridge their conceptual gaps through demonstrations of mechanical systems.

The following paper outlines a course lecture based on a demonstration titled "Soup Can Races" in which students are asked to predict the order of finish of various soup cans that are raced or rolled down an inclined plane. The goal of lecture is to address conceptual gaps surrounding students' understanding of energy-based solutions in rotational dynamics.

Demonstration setup

A 24-in. wide and 60-in. long table is typically sufficient to be used as an inclined plane when propped up 12 in. on one end. A small piece of wood (1 in. x 4 in. x 24 in.) should be used as a starting gate to assure a fair start. Little success has been achieved in trying to simultaneously release the cans by hand. A small piece of wood can just be removed in the direction down the ramp to start the cans.

The soup cans the students are asked to rank, including their masses and diameters, are shown in Table 1. The soup cans used for this demonstration were No. 1 Picnic-sized cans. For added discussion purposes a hoop and a solid disk will also be raced.

Table 1: Soup cans to be raced during in class demonstration.

Soup Can	Mass (g)	Diameter (mm)	Description
Shell of Can	39	65	A steel can with the top and bottom removed. Similar to a hoop.
Empty Can	51	65	A steel can including the top and bottom.
Crème of Concrete	835	65	A steel can with the top and bottom removed, but the intermediate space within shell of the can filled with concrete.
Tomato Soup	354	65	A can of tomato soup.
Chicken Broth	347	65	A can of chicken broth soup.
Hoop	230	100	Standard moment wheel with the mass located at the edge furthest from the axis of rotation.
Solid Disk	370	100	Standard moment wheel with mass evenly distributed between the axis of rotation and the maximum radius of the disk.

Demonstration Introduction

Before racing the cans, the students are asked to record their expected order of finish of the soup cans on the racing form provided. To aid their decision making, the following information is also given on the racing form:

1. The mass of each soup can.
2. The basic energy equation for the given system stating that the change in potential energy is equal to the sum of the translational kinetic energy and the rotational kinetic energy
3. The moment of inertia for a solid cylinder, hoop, solid sphere, and thin spherical shell about the axis of symmetry as shown in Table 2.

After recording their predictions, student are asked to discuss their order of finish with the classmates seated in their vicinity and then as an entire class. It is not uncommon to hear students argue for Crème of Concrete as the clear winner because it is heavier.

Table 2: Moment of Inertia [4]

Geometry	Mass Moment of Inertia
Solid Cylinder	$I = \frac{1}{2}mr^2$
Hoop	$I = mr^2$
Solid Sphere	$I = \frac{2}{5}mr^2$
Thin Spherical Shell	$I = \frac{2}{3}mr^2$

Racing

At this point the students have considered the problem and formulated ideas with regard to the solution. More importantly, they have vocalized their ideas to their classmates and publicly chosen a winner and a loser. The pending question lingers, "Which soup can finishes first?"

The cans are match raced in pairs to determine the order of finish. Two rubber bands are attached around the circumference of the cans at the ends to prevent them from slipping on the inclined plane. Typically students are allowed to pick the pairs for the match races. It doesn't matter as long as a final finish order is determined. The expected order of finish is shown in Table 3.

Table 3: Expected order of finish

Place	Soup Can
1	Chicken Broth
2	Crème of Concrete
3	Tomato Soup
4	Empty Can
5	Shell of Can

With an engaged classroom and several conceptual gaps mostly like identified, an in-depth analysis of the problem can help to bridge those gaps and further student understanding of rotational dynamics energy based solutions.

Analysis

Analysis of the last placed can is typically the easiest place to begin. With the Shell of Can the following assumptions are made:

1. The can starts from rest.
2. Although friction causes the cans to roll, there is no energy loss in the system due to friction.

3. The impact of air drag on the energy of the system is negligible.
4. Energy of the system is conserved
5. Shell of Can be modeled as a hoop.

The first four assumptions will be applied to the analysis of each can.

With an energy balance on the system it can be shown that the change in potential energy is equivalent to the final kinetic energy of the system. The kinetic energy of a rolling can is equivalent to its kinetic energy of rotation about its center of mass added to the kinetic energy of translation of its center of mass as shown in Equation 1.

$$mgh = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 \qquad [1]$$

By substituting the moment of inertia for a hoop from Table 2 and the relationship between angular velocity, ω, and translational velocity, v, shown in Equation 2 into the energy balance in Equation 1, the velocity of the Shell of Can determined as shown in Equation 3.

$$v = \omega\, r \qquad [2]$$

$$v_{Shell\ of\ Can} = \sqrt{gh} \qquad [3]$$

The velocity of the Shell of Can is independent of mass and radius. This can be demonstrated for the students by racing the Shell of Can against a larger and more massive Hoop.

Using the same analysis as for the Shell of Can, the velocity of the Crème of Concrete can be determined. This can is initially modeled as a uniform solid disk although density of concrete (2.4 g/cm^3) is less than steel of steel (7.8 g/cm^3). This difference will be addressed when comparing Crème of Concrete to Tomato Soup.

$$v_{Crème\ of\ Concrete} = \sqrt{1.33gh} \qquad [4]$$

Crème of Concrete can also be raced against a solid cylinder of a different size to demonstrate the independence of velocity with respect to mass and radius.

The Empty Can is a combination of the Shell of Can and two ends. A No.1 Picnic can is approximately 100 mm tall and 65 mm in diameter. Assuming the can has uniform thickness and is perfectly cylindrical, the mass of the shell can be estimated at 38.5 g and the mass of one end can be estimated at 6.25 g. Completing an energy balance on the system accounting for the rotation of the shell and the can ends separately provides the following result.

$$(2m_{end} + m_{shell})gh = 2(\frac{1}{2} I_{end}\omega^2) + \frac{1}{2} I_{shell}\omega^2 + \frac{1}{2}(2m_{end} + m_{shell})v^2 \qquad [5]$$

Simplifying using the relationship between angular velocity and translational velocity and the moments of inertia yields,

$$v_{Empty\ Can} = \sqrt{\left(\frac{4m_{end} + 2m_{shell}}{3m_{end} + 2m_{shell}}\right)gh} \qquad [6]$$

As the mass of the end of the can approaches zero for a fixed mass of the shell the results is equivalent to the result shown in Equation 3 for the Shell of Can. As the mass of the shell of can approaches zero for a mass of the end of the can the result is equivalent to the result shown in Equation 4 for the Crème of Concrete.

Given the current estimated masses for the shell and end of can the velocity of the Empty Can places fourth with a velocity,

$$v_{Empty\ Can} = \sqrt{1.07gh} \qquad [7]$$

For a more exact analysis, Crème of Concrete can then be reanalyzed using the analysis for the Empty Can assuming the concrete acts as two can ends, where the total mass of the concrete, $m_{concrete}$ = 796 g.

$$v_{Crème\ of\ Concrete} = \sqrt{1.31gh} \qquad [8]$$

Although slightly more complex, the energy balance for Tomato Soup yields,

$$(2m_{end} + m_{shell} + m_{tomato})gh = 2(\frac{1}{2}I_{end}\omega^2) + \frac{1}{2}I_{shell}\omega^2 + \frac{1}{2}I_{tomato}\omega^2 + \frac{1}{2}(2m_{end} + m_{shell} + m_{tomato})v^2 \qquad [9]$$

where the condensed tomato soup is analyzed as a solid cylinder that rotates with the shell and can ends. The velocity of the composite can of Tomato soup is,

$$v_{Tomato\ Soup} = \sqrt{\left(\frac{4m_{end} + 2m_{shell} + 2m_{tomato}}{3m_{end} + 2m_{shell} + \frac{3}{2}m_{tomato}}\right)gh} \qquad [10]$$

where the mass of the condensed tomato soup only is equivalent to m_{tomato} = 303 g.

$$v_{Tomato\ Soup} = \sqrt{1.29gh} \qquad [11]$$

Chicken Broth

Finally, one must question, “Why is Chicken Broth the runaway winner?” For Chicken Broth, it will be assumed that the viscosity of the broth is very low and that the bulk of the liquid does not rotate. Therefore, the rotational term for the broth is equivalent to zero yielding the following energy balance.

$$(2m_{end} + m_{shell} + m_{broth})gh = 2(\frac{1}{2} I_{end}\omega^2) + \frac{1}{2} I_{shell}\omega^2 + \frac{1}{2}(2m_{end} + m_{shell} + m_{broth})v^2 \quad [12]$$

$$v_{Chicken\ Broth} = \sqrt{\left(\frac{4m_{end} + 2m_{shell} + 2m_{broth}}{3m_{end} + 2m_{shell} + m_{broth}}\right) gh} \quad [13]$$

where the mass of the chicken broth only is equivalent to m_{broth} = 296 g.

$$v_{Chicken\ Broth} = \sqrt{1.77gh} \quad [14]$$

Summary of Race Results

The final velocities of each of the soup cans are summarized in the Table 4.

Table 4: Final soup can velocities summarized.

Place	Soup Can	Velocity
1	Chicken Broth	$v = \sqrt{1.77gh}$
2	Crème of Concrete	$v = \sqrt{1.31gh}$
3	Tomato Soup	$v = \sqrt{1.29gh}$
4	Empty Can	$v = \sqrt{1.07gh}$
5	Shell of Can	$v = \sqrt{1.00gh}$

The final velocity of all five soup cans is independent of the radius and mass of the cans and only a function of the combined geometric distribution of the mass in the can which is characterized by the moment of inertia for the entire can.

The Shell of Can is the slowest possible achievable velocity for a rolling object down an incline plane, because it has the maximum achievable moment of inertia.

For the Chicken Broth, the velocity of the can approaches the free fall velocity for an equivalent change in height as the mass of the shell and the mass of the end of the can approach zero.

Because each can's velocity is proportional to a constant multiplied by the square root of the distance traveled, the can undergoes constant acceleration supporting Galileo's experimental findings.

A soup can with a higher moment of inertia stores more energy in rotational kinetic energy and therefore less energy is available for translational kinetic energy, hence the can travels slower.

Applications of the Lecture

Pinewood derby cars are raced by many organizations as a way to engage youth in a fun and collaborative activity. The basic premise of pinewood derby racing is to build a car out of a block of pine wood with four wheels that rolls down a parabolic curved section and across a long flat section in the shortest amount of time. One way to increase the speed of a car is to reduce the moment of inertia of the wheels if it is allowed by the rules and therefore storing less energy in rotational kinetic energy. An additional advantage may be gained if one of the four wheels is raised up off the track 0.1 in. so that it does not rotate allowing the car to race on three wheels.

Bicycle racers have long understood the impact of bicycle weight on performance. In the year 2000, the International Cycling Union (UCI) set the minimum bicycle weight at 6.8 kg (14.99 lb). They also established rules around the types of wheels that could be used and published an approved list for non-standard wheels. Given two bicycles of equal weight, if one has wheels with a lower moment of inertia, it can be accelerated to a greater speed with the same energy input. Moment of inertia matters when races are won by fractions of a second in a field sprint. For the casual rider, it takes longer to get up to speed on a beach cruiser, compared to riding a standard road bike. The road bike is not only lighter, but its wheels have a much lower moment of inertia.

Most reciprocating engines use a flywheel to store rotational energy because the engine does not supply a continuous source of energy. Flywheels typically have a large moment of inertia which resists change in rotational speed when a torque is applied. Common applications of the flywheel include potter's wheels, car engines, punching machines, and steam engines.

Verification of Understanding

To verify their understanding and connect the lecture material to the broader course content the following question can be posed to students.

Two balls, a solid sphere and a thin spherical shell, are raced down a parabolic ramp for a fixed distance. How much longer does it take for the second ball to reach the finish line?

For each case the velocity equation as a function of height can be developed from an energy-based solution. The result can then be integrated to determine the total time after substituting the relationship between height and distance traveled. A simplified problem could be posed where the spheres are raced down an inclined plane and undergo constant acceleration.

$$t = \int_0^{t_{final}} dt = \int_o^1 \frac{1}{v(x)} dx \qquad [15]$$

Summary of Lecture

By engaging students in a class demonstration and requiring them to formulate ideas about the expected outcome, we are able to identify and bridge conceptual gaps in their understanding of engineering mechanics. Having students commit to an expected outcome helps them individually identify their own conceptual gaps and provides a motivation for their engagement in the lecture.

In this case, Soup Case Races is a creative vehicle for delivering a specific content. The same approach could easily be expanded to other ideas to help move a classroom away from a direct lecture format.

References

1. Hake, R., "Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses" American Journal of Physics, 66, 64 (1998); doi: 10.1119/1.18809
2. Smith, K. A., Sheppard, S. D., Johnson, D. W. and Johnson, R. T. (2005), "Pedagogies of Engagement: Classroom-Based Practices. Journal of Engineering Education:, 94: 87–101. doi: 10.1002/j.2168-9830.2005.tb00831.x
3. Chickering, A.W., and Gamson, Z.F., "Seven Principles for Good Practice in Higher Education," American Association for Higher Education Bulletin, Vol. 39, 1987, pp. 3–7.
4. R.C. Hibbeler, *Engineering Mechanics: Dynamics 13th Edition*, Upper Saddle River, NJ: Pearson, 2013.