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Illustrating Rotating Principal Stresses in a Materials Science Course

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Illustrating Rotating Principal Stresses In A Materials Science Course

ABSTRACT

This work constitutes a laboratory component of a junior level materials science course and illustrates the importance of rotating principal stresses in the design of components such as the automotive crankshaft. The activity is centered on Mohr's circle for biaxial stress situations involving time varying normal and shear stresses. A number of dynamic situations have been considered, namely. (a) sinusoidally varying normal and shear stresses that are in phase, (b) sinusoidally varying normal and shear stresses that are 90° out of phase, (c) constant normal stress and sinusoidally varying shear stress, and (d) sinusoidally varying normal stress and constant shear stress. Employing a graphical approach, the diameter of Mohr's circle (the absolute difference between the two principal stresses) as well as the principal stress directions is monitored. The students see that for certain stress situations the principal stress directions remain unchanged while for others the principal stress directions change with time (rotating principal stresses). In general, the size of Mohr's circle changes with time. The plotting option of the Matlab code has been employed to construct three dimensional plots for the indicated stress situations with the normal and stresses respectively in the x and y directions and time in the z-direction. The plots show how the principal directions change with time, along with the size of Mohr's circle. The students are made aware of the fact that rotating principal stresses play a very important role in designing components that are subjected to biaxial or multiaxial fatigue, such as the crankshaft. Also the diameter of Mohr's circle can be directly related to Tresca or von Mises theories of failure.

INTRODUCTION

This study constitutes a laboratory component of the Mechanics of Materials courses taught to engineering students at the sophomore or junior levels. It is important that the students learn how the external loads combine to produce stresses in a critical location of a structure or a component. This is fundamental to the understanding of the response of a structural component to a combined system of loads that result in normal and shear stresses. Mohr's circle is an invaluable tool for this purpose, especially in its ability to determine principal stresses and principal directions for combined load situations. Mohr's circle can be used to study a number of situations involving multiaxial stress states. The principal stresses can be used to evaluate material failure using appropriate failure criteria, and the nature of loading plays an important role in this process. It is desirable that the students learn the concepts of proportionality and non-proportionality of various loadings, since these are important in the design of automotive components, such as connecting rods and crankshafts. A sample problem involving combined bending and torsion of a shaft under steady and harmonic loadings is employed for this purpose. Proportional loading is defined as any state of time varying stress where the orientation of the principal stress axes does not change with respect to the axis of the shaft. Non-proportional loading is defined as any state of time varying stress where the orientation of the principal axes changes with respect to the shaft axis. The students study the "proportionality" of loadings using Mohr's circle for four specific cases for a shaft under combined bending and torsion, which are:

1. Time harmonic bending moment and time harmonic torsion that are in phase.
2. Time harmonic bending moment and time harmonic torsion that are 90° out of phase.
3. Time harmonic torsion and steady bending moment.
4. Steady torsion and time harmonic bending moment.

Although the fatigue failure is typically not addressed in Mechanics of Materials course, the students will be made aware of the fact that in many situations involving complex loadings, the locations of the critically stressed areas are not known in advance. For such cases appropriate and efficient methods are needed for fatigue analysis. The complications arise due to complex geometries and complex non-proportional loads acting on such structures.

MOHR'S CIRCLE

Transformation of stress among coordinate systems is important in structural analysis. More than 140 years ago, Mohr came up with a graphical construction (Mohr's circle) to assist with this process [Mohr, 1882]. In this paper the transformation of stresses is not specifically addressed, but the principal stresses and the associated principal directions are obtained for the four biaxial stress situations identified above.

Mohr's circle is one of the most difficult topics in Mechanics of Materials course. A number of issues appear in the area of student learning on Mohr's circle, namely,

(a) Identification of the relationship of the load on a member and the state of stress at a point.
(b) Confusion between the stress axes and the spatial coordinate axes
(c) Inability to perceive rotation of the principal axes.
(d) Relevance of Mohr's circle without reference to yield and fracture criteria.
(e) The presentation of identical information in different coordinate systems.

The procedure for construction of Mohr's circle is outlined in various texts, such as [Mott, 2008]. For the four biaxial cases considered in this paper, we essentially have a bending stress, σ_x due to bending moment and a shear stress, τ_{xy} due to torsion. The principal stresses and principal directions are given by:

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}{}^2\right]} \text{ ; And } \theta_{P1,2} = \frac{1}{2}\tan^{-1}\left[\frac{2\tau_{xy}}{\sigma_x}\right] \qquad (1)$$

PROPOSED LABORATORY EXERCISE

The students will be provided with four situations (Cases 1, 2, 3, and 4 that follow) involving steady and/or time harmonic bending moments and torsions on a circular shaft. The following example of combined loads has been specified: Two additional cases, Case 5 and Case 6 were also considered. These cases are modified Case 1 and Case 2 respectively, but include an added time varying component σy to the existing components σx and τxy.

A shaft *50 mm* in diameter is subjected to combined bending and torsion. We arbitrarily apply a bending moment of *M* and torsion of *T*. The amplitudes may be steady or time-harmonic.

We have $M = 982*f(\omega,t)$ N-m, $T = 859 *f(\omega,t)$ N-m, $d = 50$ mm, $c = 25$ mm.

The area moment of inertia, $I = \dfrac{\pi d^4}{64} = 306800 mm^4$ and polar moment of inertia $J = 613600 mm^4$

$$\sigma_x(\omega,t) = \frac{M(\omega,t)c}{I} = 70MPa \qquad \tau_{xy}(\omega,t) = \frac{T(\omega,t)c}{J} = 40MPa \tag{2}$$

The four cases will be considered next.

VISUALIZATION OF ROTATING PRINCIPAL STRESSES USING 3-DIMENSIONAL PLOTS

In order to visualize the rotating or non-rotating principal stresses, a Matlab program was employed to get three dimensional plots. Along the x and y directions the stresses were plotted and the time was plotted in the z-direction,

CASE 1: TIME HARMONIC 'M' AND TIME HARMONIC 'T' (IN PHASE)

The bending stress σ_x (in MPa), and the shear stress τ_{xy} (in MPa), are given by:

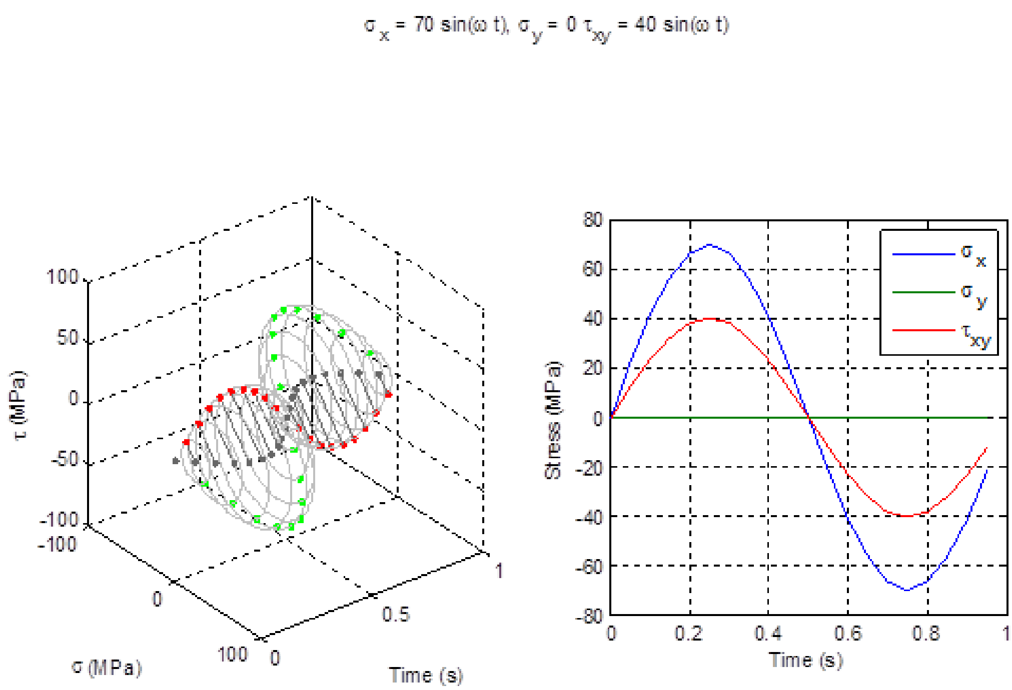


Figure 1 Case 1 Plots

It is seen from Figures 1, that the angle subtended by the principal axis is the same for all instants and is equal to 2θ = 48.8°. This is a feature of proportional loading.

CASE 2: TIME HARMONIC ‘M’ AND TIME HARMONIC ‘T’ (90° OUT OF PHASE)

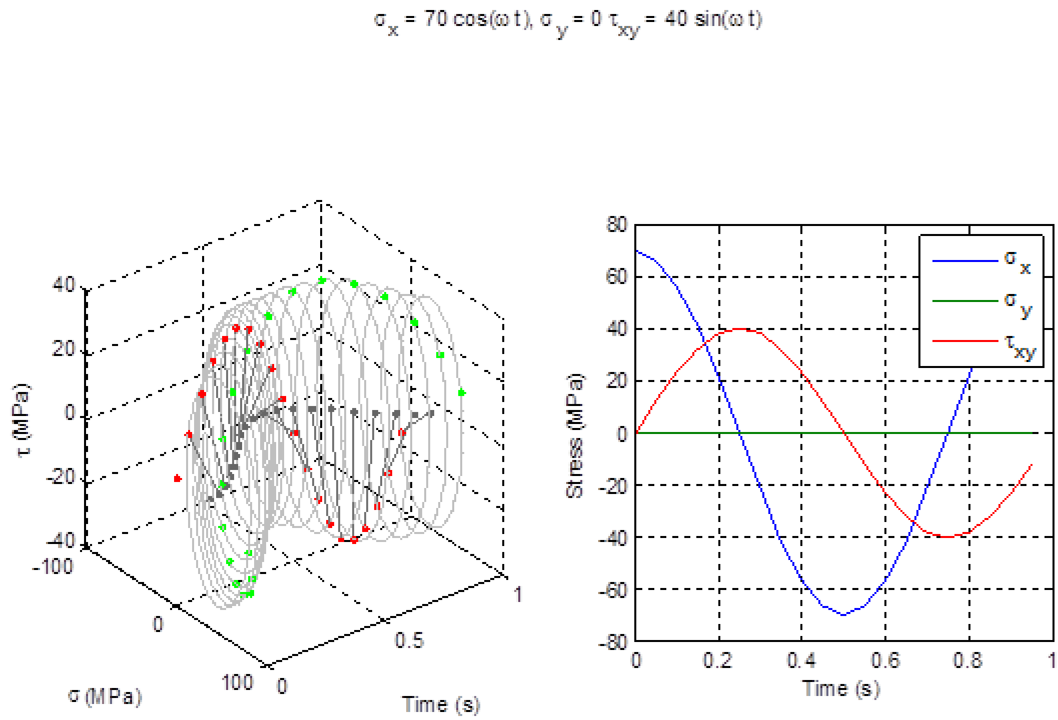


Figure 2: Case 2 Plots

Looking at Figures 2, it is clear that Case 2 is one of non-proportional loading, where the principal stress directions change with time

CASE 3: TIME HARMONIC ‘T’ AND STEADY ‘M’

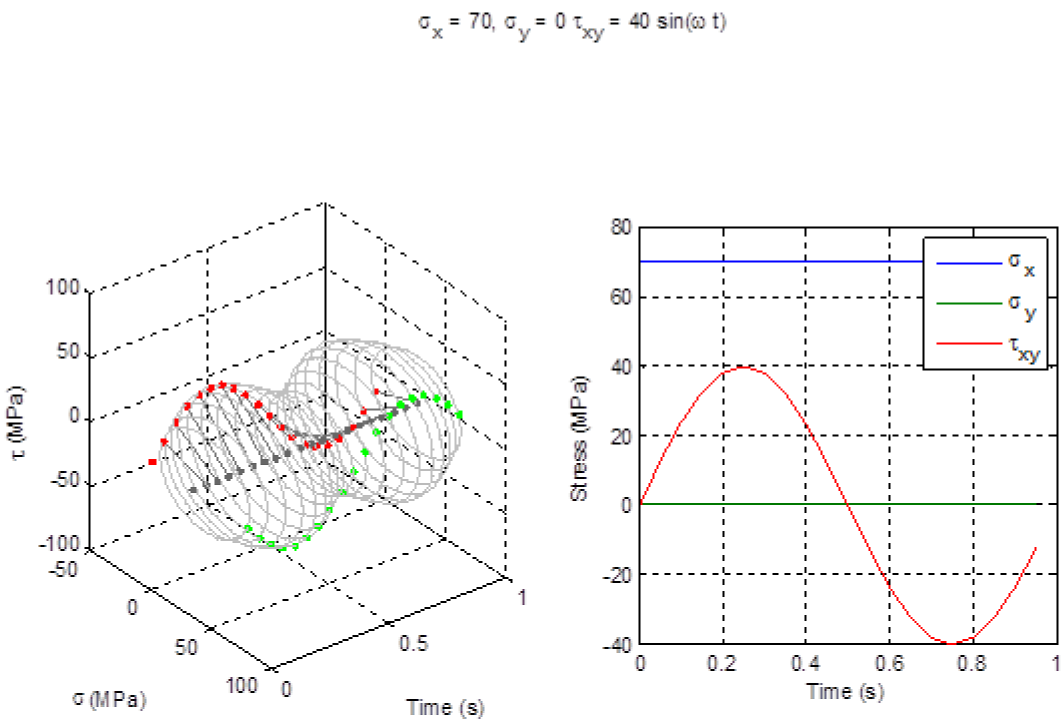


Figure 3 Case 3 Plots

Looking at Figure 3, it is evident that the case of combined loading involving a steady bending moment and time-harmonic torsion (Case 3) corresponds to a non-proportional loading where the principal stress directions change with time.

CASE 4: STEADY ‘T’ AND TIME VARYING ‘M’

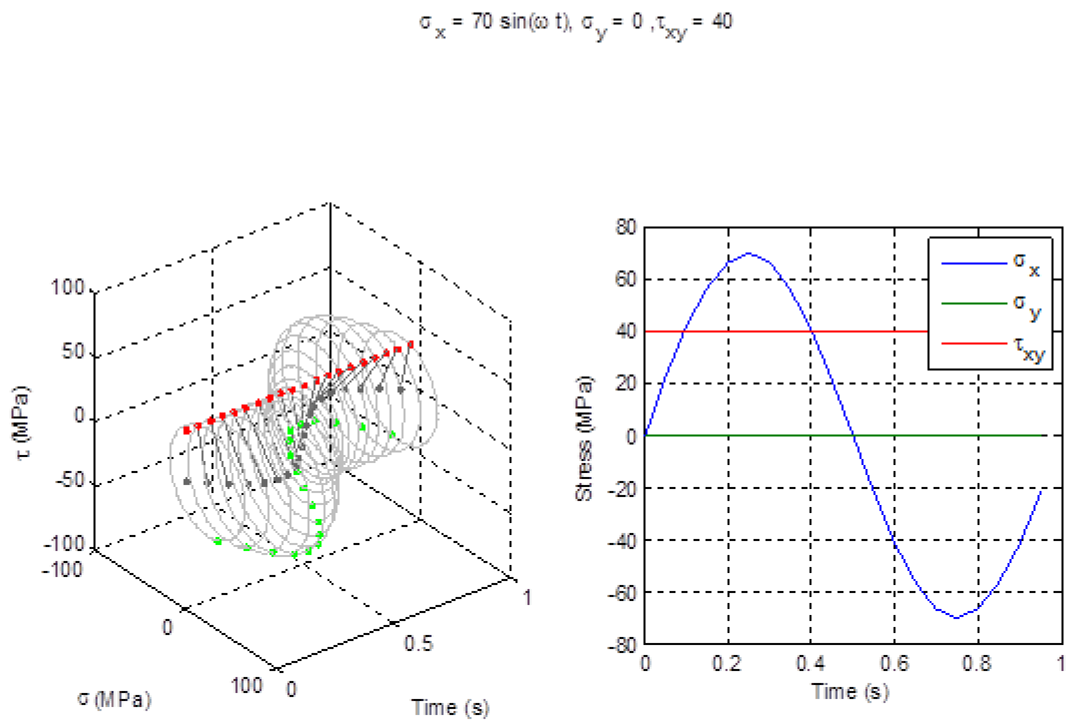


Figure 4 Case 4 Plots

The case of combined loading involving steady torsion and time-harmonic bending moment therefore corresponds to a non-proportional loading where the principal stress directions change with time.

Two additional cases, Case 5 and Case 6 were also considered.

Case 5 is the modified Case 1 and includes an added time varying component σ_y to the existing components σ_x and τ_{xy}.

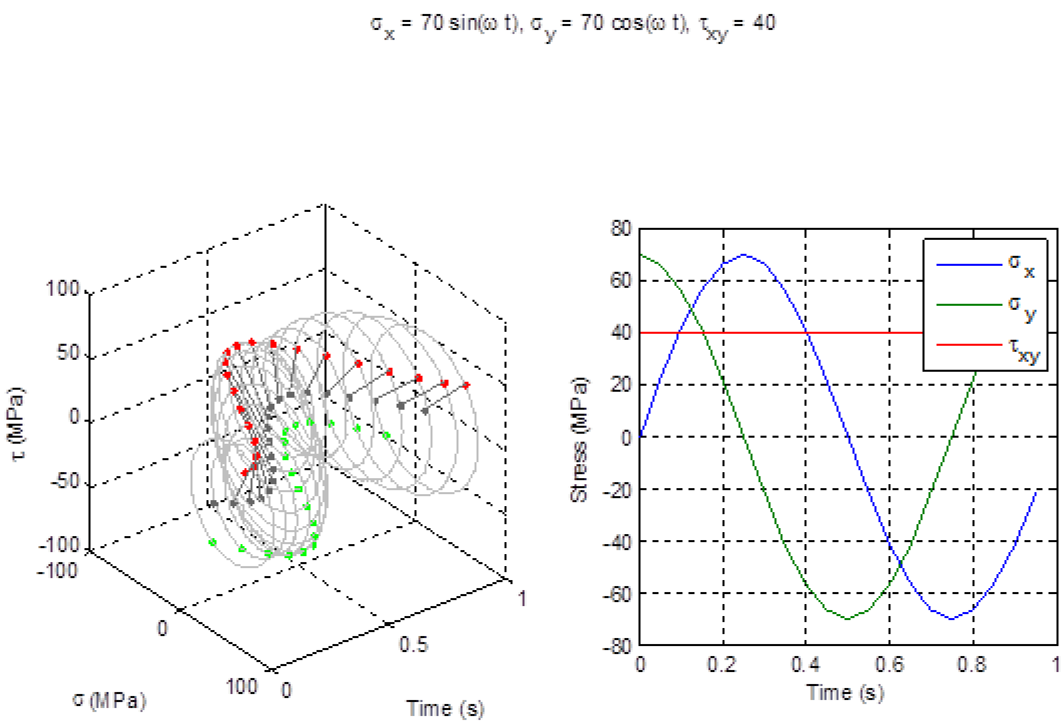
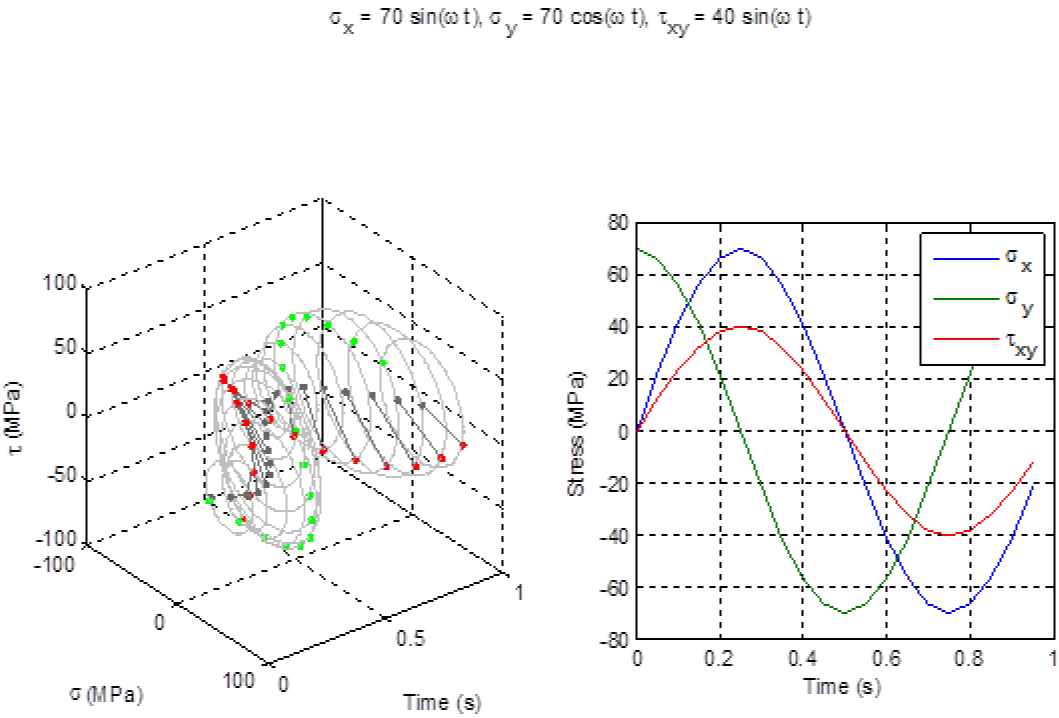


Figure 5 Case 5 Plots

Case 6 is the modified Case 2 and includes an added time varying component σ_y to the existing components σ_x and τ_{xy}.



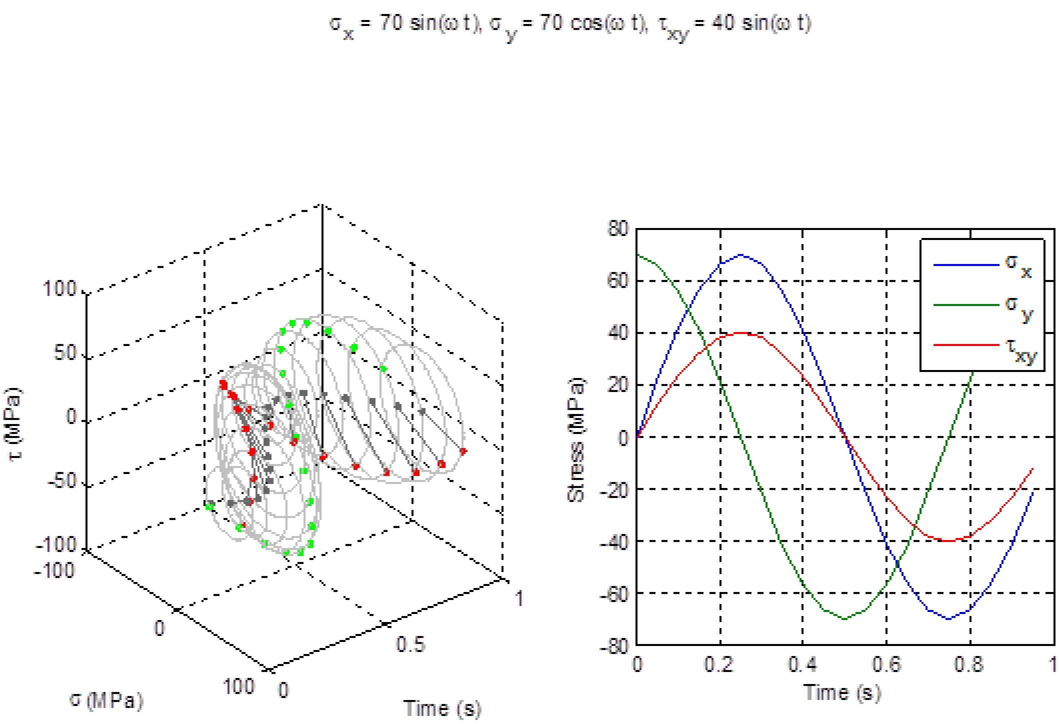


Figure 6 Case 6 Plots

OBSERVATIONS ON THE FOUR BIAXIAL CASES

Time varying bending and torsion loadings that are out of phase will always be non-proportional, while in phase time varying bending and torsion loadings will always be proportional.

(a) For the case of proportional loading as in Case 1, the size of Mohr's circle changes with time; however the principal stress directions do not change (Figure 1).

(b) For the case of non-proportional loadings, as in cases 2, 3, and 4, both the size of Mohr's circle as well as the principal stress directions change with time (Figures 2, 3, 4, as well as Figures 5 and 6)

As can be seen from Figure 1, the principal stress directions stay the same (non- rotating principal stresses or proportional loading) when the time varying normal stress and the time-varying shear stress are in phase. When the time-varying normal stress and time varying shear stress are out of phase (Figure 2) , or when one of the stress components is constant and the other time-varying (Figures 3 and 4),we have the situations of rotating principal stresses (non-proportional loadings). Figures 5 and 6 also show the same trend.

IMPLICATIONS OF NON-PROPORTIONAL LOADING

The distortion energy theory predicts that yielding occurs when the distortion strain energy per unit volume reaches or exceeds the distortion energy per unit volume for yield in simple tension or compression of the material (Budynas and Nisbett, 2011). This leads to the failure criterion that material yields when the effective stress or von Mises stress, σ' reaches or exceeds the material yield strength, S_y.

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2} \geq S_y \qquad (3)$$

Where σ_1, σ_2, and σ_3 are the principal stresses.
For the two dimensional case considered, we have $\sigma_3=0$, and thus

$$\sigma' = \left[\sigma_1^{\ 2}+\sigma_2^{\ 2}-\sigma_1\sigma_2\right]^{1/2} \geq S_y \qquad (4)$$

For Cases 1, 2, 3, and 4, static failures could be assessed using equation (4). For time-varying loadings, the fatigue failures should be considered. We could conceivably work with effective stress amplitudes, and by extending equation (14) to such situations we have

$$\left[\sigma_{1a}^{\ 2}+\sigma_{2a}^{\ 2}-\sigma_{1a}\sigma_{2a}\right]^{1/2} \geq S_f \qquad (5)$$

ASSESSMENT OF STUDENT LEARNING

The laboratory exercise was placed right after the completion of one lecture class and one homework assignment on principal stresses and Mohr's circle. The students were asked the following set of questions as a way to provide a baseline from which to measure student learning:

1. What is the purpose of Mohr's circle?
2. Sketch an example of Mohr's circle clearly labeling (a) principal stresses, (b) maximum shear stress, and (c) principal stress directions.
3. Draw Mohr's circle for (a) uniaxial tension (e.g. σ_x = 100 MPa, σ_y = 0, τ_{xy} =0), (b) biaxial tension (e.g. σ_x = 100 MPa, σ_y = 50 MPa, τ_{xy} =0) and (c) pure shear (e.g. σ_x = 100 MPa, σ_y = 0, τ_{xy} =100 MPa))

After the laboratory activity the same set of students were asked the following questions:

1. What do the rotations of the principal axis represent?
2. Under what conditions do the principal directions stay unchanged?
3. Under what conditions do the principal directions change (rotate)?
4. What can you say about the size of Mohr's circle for all the four cases analyzed?

Answers to the pre-lab and post-lab test questions were compared to identify areas of increased understanding. The principal result was that while the students could not assign the correct principal stress directions, they did quite well in how the stress magnitude varied. Over half the students in the class identified the trends for the principal stresses with varying bending moment and torsion and what the phase differences in bending moment and torsional loadings did to the principal stress directions and to the size of Mohr's circle.

CONCLUDING REMARKS

The laboratory activity provides a practice to perceive the compactness of information contained in a Mohr's circle. As such it is a unique way to study the transformation of stresses as the coordinate axes are rotated. In addition this example also provides information as to how the magnitudes and directions of the principal stresses change as a way to determine whether the loadings are proportional or non-proportional in nature. Through the activity of circle construction involving steady and time-harmonic variation of bending moment and/or torsion, the students can visualize whether the principal directions stay the same or keep varying (rotating)., and thereby establishing proportionality or non-proportionality of the applied loadings.

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