Geometric Unity Constructions

Prof. James A Wronecki, East Tennessee State University

Assistant Professor James Wronecki works in the Department of Engineering Technology, Surveying, and Digital Media at East Tennessee State University. He has a master's degree in Industrial Design from the University of the Arts in Philadelphia, Pa.

Geometric Unity Constructions

Present computer-aided-design (CAD) programs afford diverse ways to create basic elementary shapes and forms. With these programs, the three basic 2D shapes: circle, triangle, and square, and their 3D counterparts: sphere, tetrahedron, and cube may be created in a variety of ways. Selecting tool buttons, pressing various keys, and mouse drags creates/scales shapes from their center out or via diagonal selection marquees.

Most CAD techniques are proprietary, software specific, algorithmically scripted tool buttons that emphasize ease-of-use for automated rapid shape creation and fast form placement. An ideal CAD system enables the user to focus on design, and problem solving rather than on tool use. However, even the most efficient ways and effective means must induce consequentially correlated ends. As these streamlined systems tend to leave *gaps* in students knowledge base, as their sole study and use *omits* geometric construction steps that are natural and inherent to basic shape and form.

Natural, inherent geometric relationships are coherent pattern structures in *design* that encompass critical thinking, tangible perception, and applied formulations. To these *ends*, this paper outlines cognitive (*means)*, perceptual insights, and behavioral *ways* to construct geometry. The paper also introduces a Geometric Unity theory that *animates-the-origination* of the three basics basic shapes. The theory parlay's Buckmister Fuller's angular topology by tetrahedronal mensuration to inform and enlighten it's practice.[1]

A quote "The absence of limitations is the enemy of art" by Orson Welles, challenges the author to educate future artisans, designers, and technicians with guiding limitations to promote higher-order critical thinking, illuminating perceptual intuition, and expansive exercises of creative activity. Again, to these ends, we relate a Geometric Unity theory to ancient Euclidean Geometric Constructions to provide constraint-based methods to creation of basic shapes and forms. The obliging vigor of this approach binds visual thinking, form construction, and creative resolutions to the enlightening revolving of simple geometer's tools: straight-edge and compass and their modern CAD tools, 1) straight-line and 2) circle created from the center-out. The constrained challenge to construct with limited tool ways and technical geometrical means enables students to know and be able to: practically use basic CAD tools, critically think through problems with geometric constructions, and create animated expressions of the shapes of unity.

Unity Circle {1}

An animated visualization of Unity Circle's Cycle is created by a peripheral, circular sweep (temporal-constant period) of full 360° (fractional angles) orbiting around a central stable axis. In *Omnidirectional Halo*, (Fuller, pg127) our geometric unity expression is:

> "Unity is the full circle sweep around an axis. Angles are fractions of cyclic unity. Frequency means a discrete plurality of cycles within a given greater cyclic increment. Angle means a fraction of once cycle. Angle is therefore sub-cyclic-unity, while frequency plural unity. Angle is less than finite cyclic unity. Frequency is greater than finite cyclic unity."[2]

Webster's Greek term monad, stems from roots *μένειν* (*menein*), "to be stable", from μονάς (monas), "unit" from μόνος (monos), "alone".[6]

Greek Ionian System: Alphabetical Enumeration					
μ	**ο**	**ν**	**ά**	**ς**	
[m]	**[o]**	**[n]**	**[a]**	**[s]**	
Mu	Omicron	Nu	Alpha	Sigma	
40	70	50	1	200	**Σ361**

Schneider (1994) writes: "In the alphanumeric correspondences of the Greeks, the letters of the word *monas* add up to 361. The system allows for a difference of one, so the word for "oneness" becomes 360, not coincidentally, as that is also the number of degrees around a full circle".[4]

The Henagon {1} (or monogon) in Euclidean geometry is an impossible object: a self-dual polygon with one edge (360° arc) and one vertex.[6] Fuller further, considers both aspects of all systems and therefore *both* cyclical movements on a circle's *two sides* totaling angular rotation of 720°= (+ 360° *clockwise*, obverse cycle) + (- 360° reverse, *counterclockwise* cycle). The Unity Circle Cycle's front obverse and back reverse = 720°.

Unity Circle Construction

Earth's diverse centuries, cultures, and creeds represent a monad concept of oneness (unity) as a point centered in a circle. Using a geometers tools, compass and straight-edge, a unity or monad circle is created by setting compass point and "swinging-an-arc". A Unity Circle's has three physical properties or parts (tri-unity): center, radius, and circumference. Diameter (Ø) and circumference (π) are never both measurable (at the same time in similar units) as when one is in "whole" rational units the other is as an endless decimal. Pi is a circle's functional *ratio* of circumference-to diameter. Pi is the Greek 16th letter (π): it has an irrational, super-rational, and transcendental nature.[3]

Unity Sphere {1,1}

Spheres enclose the most volume with the least surface. Conceptual spheres are constructed in a 3D virtual world by a half circle, rotated 360°. Non Uniform Rational B-Splines (NURBS) are math, vector-based geometries that create, like the calculus, theoretically *infinitively* smooth surfaces. However, NURBS geometry is geodesically tessellated or triangulated to calculate the digital render. Fuller's (1960, pg 132) states our Unity Sphere is a finite, geodesic, set of interconnected points (vertices):

> There is no phenomenon "solid matter" therefore there may not be a "solid" sphere, nor a "solid" surface sphere. All spheres consist of high frequency constellations of event-points, all of which are approximately equidistant from one central event "point". All the points in the surface of a sphere may be interconnected. If most economically interconnected, they will subdivide the surface of sphere into an omni-triangulated spherical web matrix.

(Fuller pg. 128) defines geometric constructions such as the Platonic Solids as: "angularly modulated de-finite geometrical systems or figures", "independent of size", possessing "definite integrity", and are "conceptual principles of abstract thought independent of physical realization". [2] These abstract matrices are commeasurable:

> The difference between the sum of all the angles around all the vertices of any system and the total number of vertices times 360° (as angular unity) is 720° which equals two unities. The sum of the angles of a tetrahedron always equal 720°. The tetrahedron may be identified as the 720° differential between any *definite local geometrical system* (Greek Solid) and finite universe.[2] (Fuller, pg 128)

Unity Proportion
According to (Fuller, pg 126) "Unity is inherently plural. Unity is always divisible as twoness, or fourness, or sixness, of inherent minimal relationships." [2]
In (Fuller's, pg 128) Table 1. Angular Topology Independent of Size, Fuller enumerates, and illustrates the proportional relationship of Platonic Solid as being by a single, stable, tetrahedron's degrees (720°). By his formula: (finite minus De-Finite = 720°) written as: (System's Number of Vertices *multiplied* by 360°) *minus* (System's Sum of Angles *multiplied* by System's Number of vertices) = 720° and expressed below by symbolic notation as: (# of Vertices * 360°) minus [($\sum \angle$'s)* (# of vertices)] = 720°.

Unity Triangle {3}

The Greek *trigon* "three sided" is the tri-angle. A triangle's three vertices and segments creates a stable structure whose synergistic super strength is greater than the sum of it's segmented parts. The sum of the angles of any planar triangle =180° always.

The Schläfli symbol {} defines an object in terms of itself. [6] Schläfli, a Swiss geometer, introduced concepts of higher-dimensional spaces and multidimensionality.
Where {e} is to # of a e-gon's {# of e-edges} as tri-gon (e3) is to tri-angle {3}; and {e, f} is to e-gon's {# of e-edges, @ each e-vetex's # of adjacent f-faces}; as tetrahedron {3, 3}.

In contrast to Unity Circle, triangle encloses the smallest area with greatest perimeter. Three front faces of an equilateral triangle's angles equal 180° or one-half cyclic unity.

As all triangles have two faces, their obverse and reverse, the angles of both faces of a triangle add up to 360° or one Unity Circle Cycle. Therefore, the angles or fractions of cyclic unity comprising the front and back faces of an equilateral triangle provide an accurate angular representation of unity 360°= -180° back reverse + 180° front obverse.

Triangle Construction
In geometry an equilateral triangle may be constructed from two Unity Circles.

A geometric way to construct an equilateral triangle begins with a Unity Circle centered at A. At any point along the circle's circumference, we swing-an-arc of equal radius to create a 2nd Unity Circle @ B.
{3} is in twin {1}'s connect ABC.

Fig. 1a: Unity Triangle

The overlapping space between the two circles is called a *vesica piscis,* latin for *bladder-of-fish*. In ancient Maya the glyph for *conjure* is a hand holding a fish, as to pull from the water (consciousness).
These circles' center-points are connected by the 1st straight-line segment AB.
From points A & B, lines can be draw to C, the tip of the vesica pisces to create an inner smaller equilateral triangle above AB. The Unity Triangle is in to two Unity Spheres.
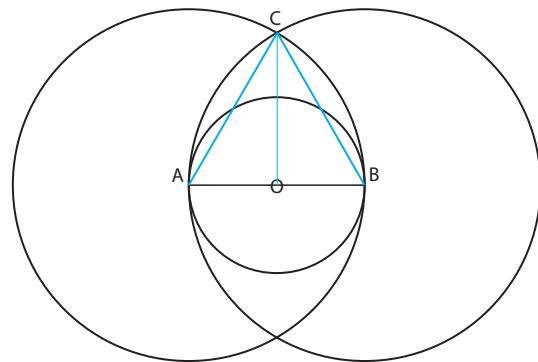
Two Unity Tetrahedron: 720° {3, 3}

Two Unity Tetrahedron:

The tetrahedron is comprised of *four* equilateral triangular *faces*. The tetrahedron is the *only* three dimensional shape whose vertex corners are equidistant. It is the minimum *four-vertex*, *four-sided* structure in universe and as such is referred to as "simplex".

As the sum of the angles of a planar triangle is always 180° so the sum of all the angles of a tetrahedron, regular or irregular, is always 720°.

A tetrahedron equals two unities.

Tetrahedron Construction
The equal angles in a tetrahedron's four vertices =180°. Each vertex angle = 60°. A 2D tetrahedron construction, we 1st follow the steps outlined above in fig1a. Create two Unity Circles so that their *centers* are at the other's *circumference*.

Fig. 1b: Two Unity Tetrahedron:

In Fig 1b: Two Unity Tetrahedron: the initial smaller triangle's edges are extended to the *circumferences* to points D & E. Connect DE to create an outer larger equilateral triangle.

The center of the larger equilateral triangle labeled C, is at the intersection of lines AE and BD drawn from the midpoint of the triangles edges A & B to the triangles vertices D & E. A 3D tetrahedron may be created by folding (rotating) the construction. Fold visible smaller triangular faces along blue lines AB, BG, and AG. The tetrahedron demonstrates Fuller's concept of Synergy as four sides are stronger than sum of parts.

Unity Square {4}

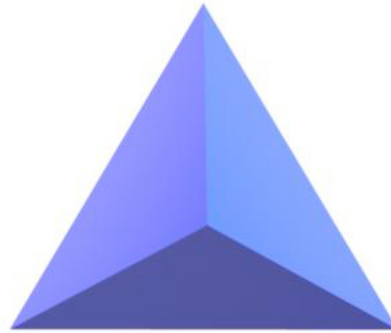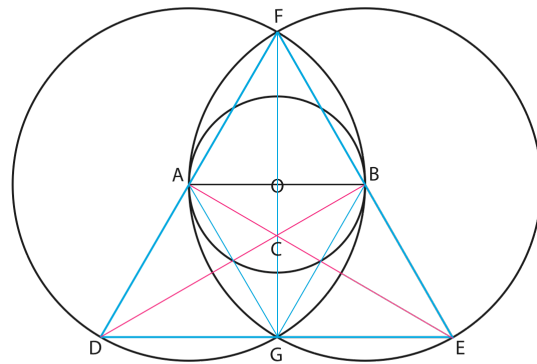A square is a plane figure of four equal sides and four right 90° angles. Total angles in a square 4 x 90°= 360° and therefore are one Hexangular Unity Cycle.

A squares four parallel/perpendicular lines and equal angles creates structures that are straight, right, and even. The term square also numerically denotes the multiplication of a number by itself.

The squares unity is divided through the drawing of a square's diagonal. The side of the original square is it's "root", and may be given a value of one. The square root may be a number that when multiplied by it's self will produce a given number or quantity such as 1*1= 1, 2*2=4, 3*3=9).

Square Construction

In Fig. 3: Unity Square: We rotate two Unity Circle Cycles to employ the vesica pisics. The above steps are added for visual continuity but are not needed to circumscribe a square in a circle. Fig. 3b. Illustrates below without the above. Draw any line OA and locate any point C to the right of OA and somewhere near its midpoint. In this diagram C is positioned @ C. With center C and radius OC swing an arc of at least half a circle, cutting OA @ B. Join O & C continuing the line until it cuts the green arc @ D. Thus square, parallel and perpendicular lines are drawn OB:BD. With centres B & E swing two arcs with radius OB until they intersect @ D. Draw square OBDE.
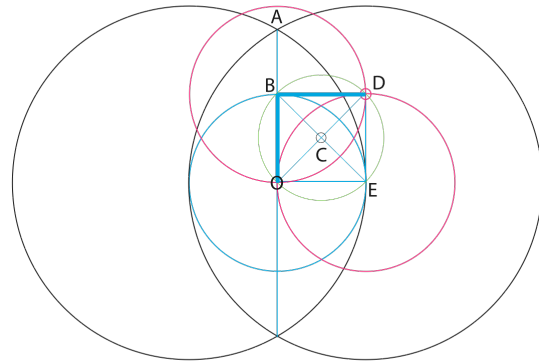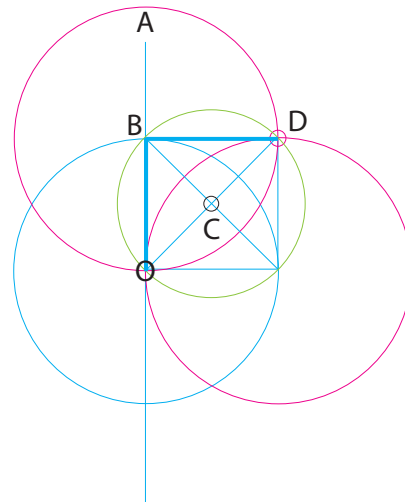
Fig. 2b: Unity Square:

Square Root Construction

Fig 2B: Unity Square Root construction follows the previous methods by creating a vesica piscis with a circle inscribed within. Draw parallel and perpendicular lines or follow fig 2a above to swing arcs to find points A and C. Draw square ABCD. The side of a square is called it's root (√). The initial or primary smaller square OB (square 1) is √1. The side AB of the secondary unity square ABCD is √2.

Fig. 2b: Unity Square Root
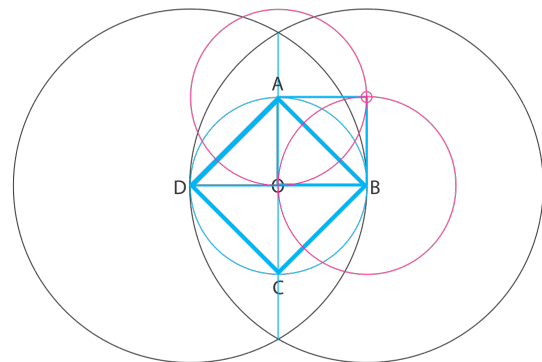
Conclusions

The geometric unity constructions outlined in the paper provide students with an introduction to the physical properties, proportional relationships, or mathematical attributes of three basic shapes the unity: circle, triangle, and square. This paper provides a foundation for further study and application of the shape archetypes. The geometric unity theory introduced in this paper and their corresponding geometric unity constructions provide a basis for an introductory unity on geometry creation. These lessons may be applied in the classroom and in doing so activate multiple intelligences by creating these archetypes with compass and straightedge, and their digital CAD counterparts. The procedural development of the shapes creation with the geometric vesica pisces promote geometric thinking and doing. Fundamental engineering design graphics inherent in this discourse include: arcs, intersection, tangency, quadrant, bisection, perpendicular, parallel etc. The presentation demonstrates how this method can be applied to create animated visualizations of unity.

Bibliography
1. Fuller, B (1971) No More Secondhand God and Other Writings Garden City, NY Doubleday & Co. Inc.
2. Fuller, B (1960) Omni-Directional Halo Garden City, NY Doubleday & Co. Inc.
3. Lawlor, L (1982) Sacred Geometry Philosophy and Practice Thames & Hudson Ltd. London
4. Schneider, M (1994) The Beginner's Guide to Constructing the Universe Harper Perennial NY, NY
5. Websters New World Dictionary of the American Language, College Edition (1953-1960) The World Publishing Co. Cleveland and New York
6. Wikipedia New http://www.wikipedia.org