2021 ASEE ANNUAL CONFERENCE
Virtual Meeting | July 26–29, 2021 | Pacific Daylight Time
Paper ID #32775

Are you sure about that? Introducing Uncertainty in Undergraduate Engineering

Sophia V. Yates, Smith College

Sophia V. Yates is a junior studying engineering sciences in the Picker Engineering Program at Smith College. She is interested in structural engineering and engineering education. Yates is planning on pursuing a graduate degree in STEM education after her time at Smith College.

Dr. Christopher H. Conley, Smith College

Chris Conley is currently a research affiliate with the Picker Engineering Program at Smith College. He has served on the faculties of four institutions over the last three decades. He has also worked as a researcher at four federal laboratories. His research interests include modeling and simulation, both physical and numerical, with special interest in the response of structures to extreme loads. Helping others learn to apply engineering mechanics to better understand their world is a passion.

Dr. Aaron J. Rubin, Smith College

Aaron J. Rubin is a lecturer at Smith College where he teaches Junior and Senior level undergraduate engineering courses including Finite Element Modeling and Senior Design Clinic.

Are You Sure About That?

Introducing Uncertainty in Undergraduate Engineering Education

Abstract

George Box famously stated “all models are wrong, but some are useful” in a 1978 workshop on Robustness in Statistics. Building on that notion, we explore how undergraduate engineering students may begin to learn to differentiate the "useful" models and the limits of their usefulness. In the last forty years, the field of engineering has become increasingly reliant on computer-based modeling. Usage of numerical modeling and simulation in the undergraduate engineering curriculum has also expanded. The increase in usage of this powerful tool is not without consequences. Engineers often neglect the potential impact of the simplifications that go into creating a model and thus neglect the uncertainties inherent in simulation results. This high faith in models can lead to incorrect conclusions and potentially catastrophic consequences.

In this paper, the authors discuss the necessity and utility of learning about uncertainty in conjunction with modeling and simulation at the undergraduate level. They provide content to scaffold a student’s intuition about uncertainty while also enhancing the student’s understanding of modeling in multiple disciplines. The authors use a case study that compares a finite element analysis (FEA) simulation to data from physical experiments as a foundation for learning about modeling and simulation. Comparing physical experiments and results of numerical simulations, the students will investigate how the results were affected by assumptions. Quantifying the impact of their assumptions through analysis of uncertainty will accompany the validation of their simulations.

Based on the literature reviewed, the current treatment of uncertainty in numerical modeling follows the general form of the quantification of uncertainty in physical modeling. Therefore, the lesson content on quantification of uncertainty reviews what is commonly covered when discussing physical experiments, and demonstrates how that can be extended into the numerical modeling realm. Both Type A and Type B evaluations of uncertainty are covered. Incorporated throughout the educational tools presented in this paper is a vocabulary necessary to discuss computer-based numerical simulations.

Introduction

Engineers are constantly incorporating new technologies in their work. While this may seem obvious given that engineers are responsible for the creation of a significant portion of the technologies used in the world, the practicing professional engineering community is sometimes conservative in their embrace of new technologies. This was the case with finite element analysis [1], [2], but with vastly improved user interfaces and local availability of significant computing power the use of high-fidelity numerical simulations is seeing significant growth both in practice and in the classroom.

In Froyd, Wankat and Smith’s paper “Five Major Shifts in 100 Years of Engineering Education” [3] they include “Simulations” as part of the “Fifth Major Shift” and note that “Simulation…has become ubiquitous in engineering education.” More recently, Magana [4] presents research done with a panel of 18 experts from academia and 19 from industry, on what modeling and simulation (M&S) practices should be integrated into engineering education. There was significant consensus on the need for skills related to validation, acknowledging uncertainty in the interpretation of simulation predictions, and developing intuition and being critical of results. Developing a healthy doubt of computer-generated results in students is an issue that others have addressed [5], [6]. Both the increasing use of M&S, and the need for the credibility of M&S results to be questioned, is addressed in the ASME *Guide for Verification and Validation in Computational Solid Mechanics* [7].

While a lack of understanding of, and emphasis on, uncertainty in numerical simulations may not present immediate problems in undergraduate education, in professional practice the potential for catastrophic consequences is clear. This was the case in the early morning hours of January 18, 1978 when the roof of the then Hartford Civic Center collapsed. An image of the failed structure is shown in Figure 1. Fortunately, no one was injured. This outcome would likely have been

Figure 1. View of the Hartford Civic Center roof, January 1978 – Connecticut Historical Society (from [8])

very different had the roof collapsed hours earlier when nearly five thousand fans were watching a collegiate basketball game. Investigations of the failure [9], [10] report that deflections during construction were measured to be twice those predicted by the computer analysis, and that some compression members in the completed structure had unbraced lengths of 9.14m (30ft) rather than the 4.57m (15ft) used in the computer model.

Having too much faith in the results of computer-based analyses was also found to be a major contributor to the failure of the pedestrian bridge at the Florida International University on March 15, 2018, shown in Figure 2. This failure was thoroughly investigated by the National Transportation Safety Board (NTSB) [11]. The NTSB notes that the engineers' "modeling for the bridge design resulted in a significant underestimation of demand at critical and highly loaded nodal regions." [11, p.100]. They also noted in their report that "beginning with the cracking identified on February 24, 2018, the distress in the main span structure was active, continued to

Figure 2. View of the collapsed FIU pedestrian bridge on March 15, 2018 (from [11])

grow, and was well documented by all parties involved in the design, construction, and oversight of the bridge." and that "the rate of premature concrete distress was clear evidence that the structure was progressing toward failure…" [11, pps. 101-102]. Unfortunately a construction worker and five occupants in vehicles under the bridge when it collapsed, died.

These are but two examples of undue faith being placed on results of simulations, even in the face of the real-world systems exhibiting behavior contrary to the simulation results on which their design was based. The goal in writing this paper is to highlight the need for inculcating an appropriate skepticism in engineering undergraduates regarding modeling and simulation results.

This is a work in progress. We are still trying to better assess how many engineering undergraduates are introduced to uncertainty quantification (UQ) as regards physical experimentation and computational simulations. We are also seeking best practices for incorporating UQ into undergraduate engineering programs.

Background

The latest edition of the ABET criteria [12] includes as a student outcome for all engineering programs "an ability to develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgment to draw conclusions." We believe that ABET is referring to physical experimentation, but we see no reason that this student outcome should not be interpreted to include computational experiments. Others are also taking this view of experimentation [13], [14]. ABET also includes as a curriculum criteria for all engineering programs "a culminating major engineering design experience that 1) incorporates appropriate engineering standards and multiple constraints, and 2) is based on the knowledge and skills acquired in earlier course work." The literature indicates that many of these design experiences are being completed with limited attention being paid to uncertainty.

We will use the terms model, modeling and simulation as applying to both physical experiments and computational experiments. The common definitions of verification and validation can also be applied to activities in both the physical and computation realms, but these terms have taken on somewhat specialized meanings in regard to computational simulations. In the computational realm [7], [15], [16], [17] **verification** is considered as having two components. The first has to do with ascertaining the accuracy of the numerical algorithm representing the mathematical model, and the second with ascertaining the solution accuracy, which is often influenced by the level of discretization in the finite element model. To help put these terms in context we will use the case study that is outlined in Figure 3.

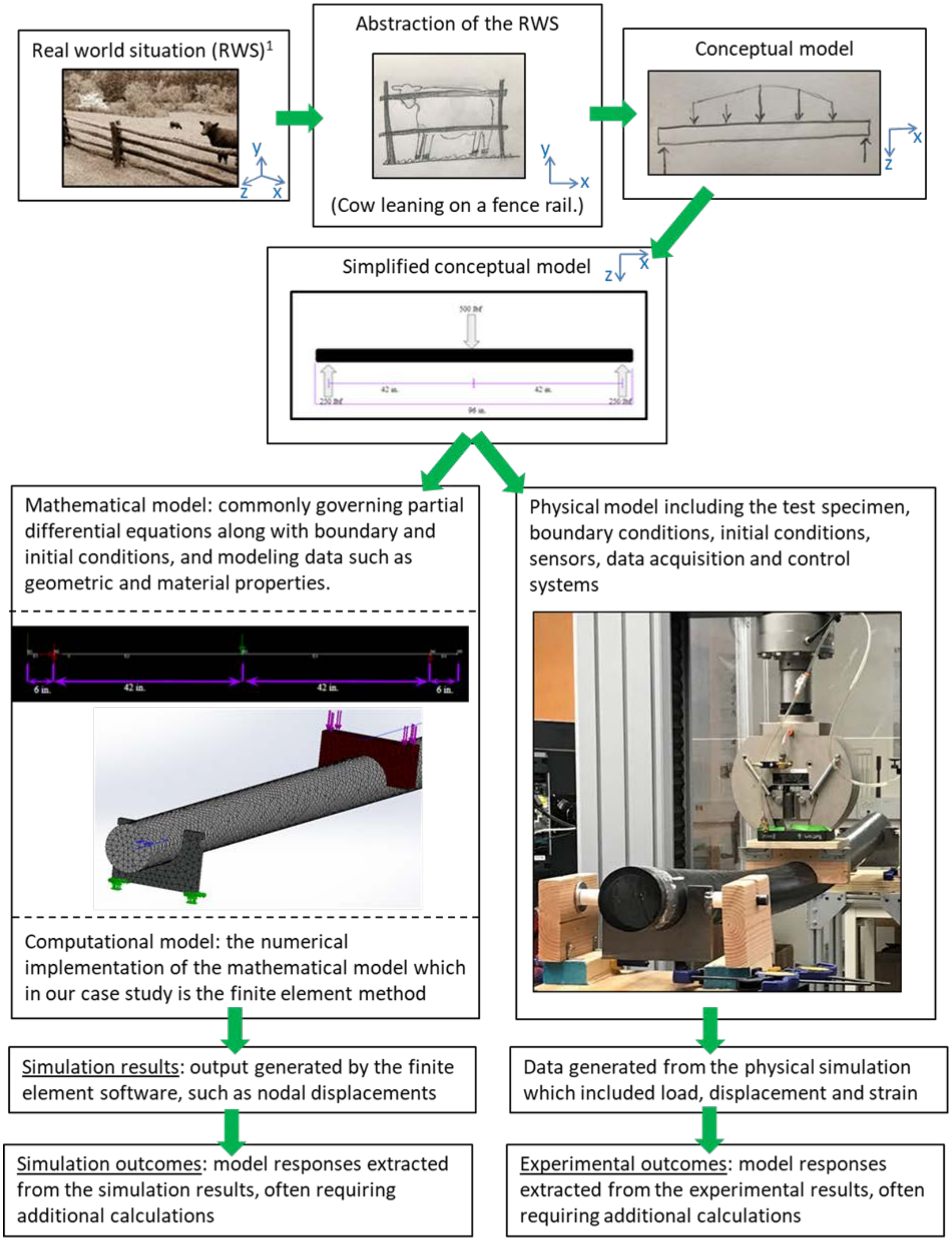


Figure 3. Steps in the computational and physical modeling and simulation processes

[1] https://fineartamerica.com/featured/cow-looking-over-split-rail-fence-on-the-blue-ridge-parkway-john-harmon.html

Our case study comes from an undergraduate research project that was not part of a formal course. A local manufacturer wanted to investigate the possible application of a product in a new market. They had the capability to manufacture 8ft long 4in diameter solid rods from a blend of recycled polyethylene (PE) and polypropylene (PP), and wanted to explore the use of these rods as fence rails in agricultural applications. Developing a polymer blend was motivated by PP having higher inherent stiffness but somewhat limited availability, whereas PE is much more readily available as a recycled feedstock.

Figure 3 shows the real-world situation (RWS), along with the steps taken to develop a laboratory experiment to determine the flexural stiffness of the rods in a fence application. As is the case in most laboratory investigations, several assumptions and approximations are made in the process of working towards an experiment that can be practically accomplished, while retaining what are believed to be key features to obtain a useful result. We also show the steps involved in analyzing the RWS using finite element analysis. In addition to emphasizing the uncertainty associated with both physical and computational experiments, we hope to show the synergy between the two approaches

The definition of **validation** used in the computational literature [7], [15], [16], [17] is in line with the common definition, but specifically refers to the comparison of the two end steps in Figure 3. Validation experiments are physical experiments designed and conducted for the specific purpose of validating computational models and simulations.

For our purposes here we adopt the NASA definition of **uncertainty:** "The estimated amount or percentage by which an observed or calculated value may differ from the true value." [16, p.15] The first instance of a discussion of the propagation of uncertainty in experiments we found was in *Testing and Inspection of Engineering Materials* by Davis, Troxell and Wiskocil (1941) [18]. More complete discussions are presented by Baird [19] and Taylor [20], these being texts that engineering undergraduates may have been introduced to in conjunction with a physics laboratory course.

In the latest version of the FE Reference Handbook (FERH) [21], the Kline-McClintock (K-M) equation is presented for use in determining an estimate of the uncertainty in a calculated result R, and has the form of equations presented by other authors [19], [20]. Unfortunately, the K-M equation is not presented in the majority of undergraduate engineering text books we have reviewed, which may indicate that the propagation, and quantification, of uncertainty is not being generally addressed.

In their 1953 paper, Kline and McClintock [22] note the need for the development of standards for description and analysis of uncertainty in single-sample experiments. The international standards that Kline and McClintock promoted now exist, to a degree. Standard language related to uncertainty in measurements is addressed in the International vocabulary metrology - Basic and general concepts and associated terms (VIM) [23]. There is a companion standard produced by the Joint Committee for Guides in Metrology, entitled Evaluation of measurement data - Guide to the expression of uncertainty in measurement [24], that is commonly referred to as the "GUM" in which the propagation of uncertainty is thoroughly covered.

The latest edition of the FERH is now referring to the VIM for definition of terms, and the version of the K-M equation presented in the "Engineering Probability and Statistics" section of the FERH [21, p.69] is essentially the same equation presented in the GUM [24, p. 19]. But that form does not address single-sample experiments. The GUM presents two types of evaluation of standard uncertainty, Type A where the components of the result(s) are evaluated from repeated observations; and Type B where components are evaluated by other means, including judgment [24, sect. 4.3]. The second form of the K-M equation in the FERH [21, p.226] is not written in terms of standard uncertainty as noted in the GUM, but we feel this form of the K-M equation is an expedient way to address uncertainty in many if not most applications in undergraduate engineering education, where single-sample experiments, both physical and computational, predominate. Further, it can be readily used with students who have covered the calculus, but may not have taken a course in probability and statistics.

Returning to our case study, it was quickly realized that physically applying a non-uniformly distributed load in the lab was not practical and the concept of the loading of the assumed to be leaning cow had to be simplified. As shown in the simplified conceptual model in Figure 3, it was decided to model the cow as a point load in the center of the rail. The physical model shown in Fig. 3 was then constructed. The cradle supports on the ends of the beam were built to simulate roller supports, and the center load was applied by a universal testing machine (UTM) through a custom-built loading saddle. The UTM measured cross-head displacement and load simultaneously. A strain gage was applied to the bottom of three of the beams at their center.

Given the goal was to characterize the stiffness of the three polymer blends, a data reduction equation (DRE), or "derived quantity" [23], was required. The intent of the design of the experiment was a simply-supported beam with a point load at its center. The standard deflection equation for this type of beam support and loading [21], based on Bernoulli-Euler beam theory, was rearranged to solve for the elastic modulus, with the result shown in Equation 1.

$$E = \frac{PL^3}{48\delta I} \qquad (1)$$

Load is represented by P, the span of the beam by L, the centerline displacement by δ, and the area moment of inertia by I. In order to use this equation in our estimation of uncertainty, the area moment of inertia must be put in terms of the measurable diameter, D:

$$E = \frac{PL^3}{48\delta I} = \frac{PL^3}{48\delta \frac{\pi(\frac{D}{2})^4}{4}} = \frac{4PL^3}{3D^4\delta\pi} \qquad (2)$$

Uncertainty Estimation in Physical/Material Experimentation

Figure 4 outlines what we see as the major sources of uncertainty in our experimental outcomes. There are several sources of uncertainty in moving from the RWS to the simplified conceptual model. While those are likely significant in addressing the suitability of the polymer rods as rails in an agricultural fence, they are outside the realm of this study, provided the client agrees that the research question is which polymer blend, of the three being addressed, has superior stiffness when acting as a simply supported beam with a point load at its center.

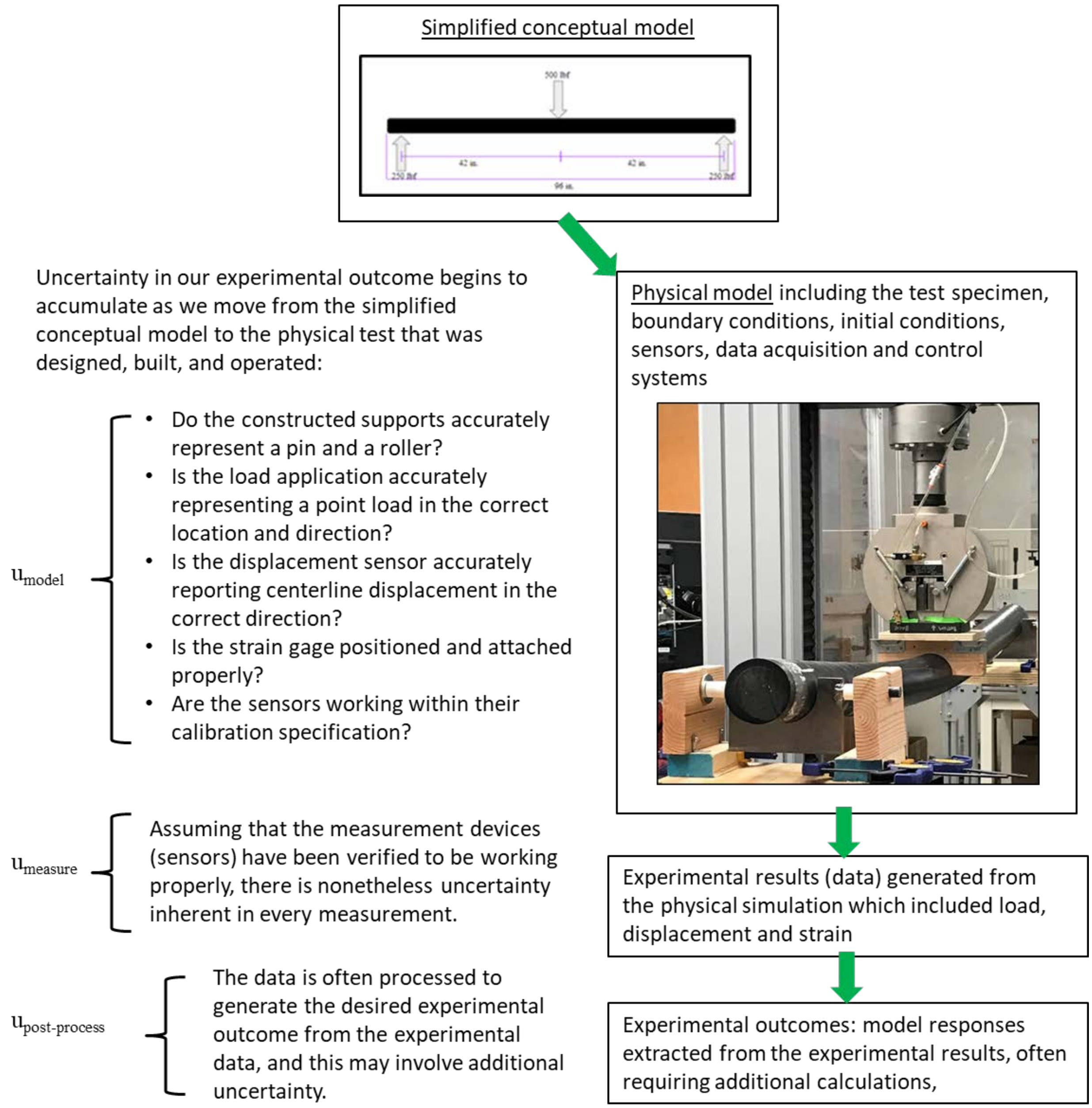


Figure 4. Sources of uncertainty in the experimental outcomes

In Figure 4 we have grouped sources of uncertainty into the three components: u_{model}, $u_{measure}$ and $u_{post\text{-}process}$. The general intent of the categorization is to promote contemplation and discussion of sources of uncertainty. These categories also facilitate an equation for the uncertainty of an experimental outcome as being the sum of the uncertainties from all sources, as in Equation 3.

$$u_{outcome} = u_{model} + u_{measure} + u_{post-process} \tag{3}$$

The literature often relates uncertainty to error, and categorizes errors as being random or systemic. Further, uncertainty is often categorized as being either aleatoric or epistemic. The art and science of uncertainty quantification continues to evolve. While being introduced to these concepts fits in well with getting students to contemplate and discuss uncertainty, we are striving for a simple introductory approach to uncertainty quantification (UQ) that we feel Eqn. 3 represents.

For our case study, much of the uncertainty categorized into $u_{measure}$ is reflected in the use of our data reduction equation (Eqn. 2). We will use the DRE to quantify the uncertainty in our measurements and thus estimate $u_{measure}$.

Our estimates of the uncertainty associated with each of our measurements to be used as input to the DRE is noted in Table 1. To come up with these values we reviewed the calibration data for the load and displacement sensors, and critically reviewed how the diameter and length of each specimen was measured

Table 1: The experimentally measured values and their associated uncertainties

Measurement	Value	Uncertainty
P = load = x_1	500.0 lbf	± 2.5 lbf = w_{x_1}
L = beam span = x_2	84.0 in	± 0.5 in = w_{x_2}
δ = beam ℄ displacement = x_3	2.64 in	± 0.25 in = w_{x_3}
D = beam diameter = x_4	4.00 in	± 0.25 in = w_{x_4}

Substituting the data in Table 1 into Equation 2, E is determined to be 186103 psi. Following the general rule for significant figures in calculations involving multiplication and division as presented in the FERH [21, p.2], this result is reported as: E = 186 ±1 ksi. References [20] and [25] provide more information on significant figures for those interested.

The Kline-McClintock equation as presented in the "Instrumentation, Measurement, and Control" section of the FERH [21] is: "Suppose that a calculated result R depends on measurements whose values are $x_1 \pm w_1$, $x_2 \pm w_2$, $x_3 \pm w_3$, etc., where $R = f(x_1, x_2, x_3, \ldots x_n)$, x_i is the measured value, and w_i is the uncertainty in that value. The uncertainty in R, w_R, can be estimated using the Kline-McClintock equation:"

$$w_R = \sqrt{\left(w_1 \frac{\partial f}{\partial x_1}\right)^2 + \left(w_2 \frac{\partial f}{\partial x_2}\right)^2 + \cdots + \left(w_n \frac{\partial f}{\partial x_n}\right)^2} \quad (4)$$

Table 1 identifies the variables in Eqn. 4. The uncertainty in our measurement, $u_{measure}$, is represented by w_R.

Using the Kline-McClintock equation (Eqn. 4) with the data produces the following:

$$w_R = \sqrt{(\frac{4L^3}{3D^4\delta\pi})^2 2.5^2 + (\frac{4L^2P}{D^4\delta\pi})^2 0.5^2 + (\frac{-4L^3P}{3D^4\delta^2\pi})^2 0.25^2 + (\frac{16L^3P}{3D^5\delta\pi})^2 0.25^2} \quad (5)$$

$$w_R = 49871\ psi = 49.9 ksi \tag{6}$$

In this case, the experimental outcome would be reported as: E = 186 ± 50 ksi

Comparing the two approaches, the uncertainty using general significant figure rules is roughly 1%, (rounded to the nearest whole percent) whereas using the K-M equation the uncertainty is roughly 27%. This seems large, and presents a segue into how UQ benefits experimental design.

What if a set of calipers had been used to measure the diameters of the beams, and what if the outer surface of the beams was not so irregular? Going back into the K-M calculation, with the uncertainty in the diameter measurement, w_{x_4}, set to 0.0625 inches, the measurement uncertainty, $u_{measure}$ (w_R) becomes 21396 psi, and now represents an 11% error. The utility of UQ in experimental design is of course well documented in the literature [20], [26].

Given an estimate for $u_{measure}$ we still require estimates for u_{model} and $u_{post\text{-}process}$. Addressing the easy one first, two people performed the calculations to determine E independently. All other calculations were also checked in a similar fashion. While this does not totally eliminate all uncertainty, we judge the uncertainty in our post-processing of data ($u_{post\text{-}process}$) is negligibly small relative to the other two terms and can be ignored. Addressing u_{model} presents a much greater challenge.

Our physical model was built with the intent of using the data reduction equation, Eqn. 1, which is based on Bernoulli-Euler beam theory. The assumptions implicit in using Eqn. 1 include: the beam being composed of a homogenous and linear-elastic material, the beam being prismatic and symmetric about the axis of loading, the beam's displacement being small enough to ignore geometric nonlinear effects, that plane cross sections remain plane and normal to the neutral surface, that shear deformations can be ignored, and that the beam is simply-supported.

The experimental data clearly demonstrates the polymer beams were behaving in a nonlinear fashion. The beams appeared to be homogenous and prismatic, and being circular, their cross sections should be symmetric about the axis of loading. Given our data, we cannot address geometric-nonlinear behavior, nor shear deformations, as being insignificant. Finally, we cannot address how well a pin-and-roller condition was achieved, nor how well a point load was modeled. In short, we can't quantify u_{model} based on the data collected.

In reflection, some of the uncertainty in our physical model could have been quantified by testing a beam of a "known" material, such as an aluminum pipe, that could more confidently be considered as homogenous and prismatic, and linear-elastic for the load range of interest, with a "known" E. If such a beam had been tested in our setup, and the results matched beam theory, we could estimate uncertainty associated with the cradle supports and the saddle loading. Additional instrumentation would be needed to address the significance of shear deformations.

To summarize: u_{model} is greater than zero, but currently unknown; u_{meaure} = 50 ksi (but could have been reduced with better measurement data); and $u_{post\text{-}process}$ is negligible. Therefore: $u_{outcome} = u_{model} + u_{measure} + u_{post\text{-}process} > 50$ ksi.

Uncertainty Estimation in Computational Solid Mechanics

Figure 5 outlines our proposed characterization of the major sources of uncertainty in computational simulation outcomes. As was done for the experimental process tree, the uncertainty in progressing from the RWS to the simplified conceptual model is not addressed. The uncertainty in our simulation outcome begins to accumulate as we move from the simplified conceptual model to the mathematical model, and continues to accumulate as we work towards the outcome(s).This summary does not reflect the extensive and ongoing research and developments in uncertainty quantification for computational simulations over the past several decades, but we are striving for an approach to UQ that a typical engineering undergraduate could understand, appreciate and potentially utilize.

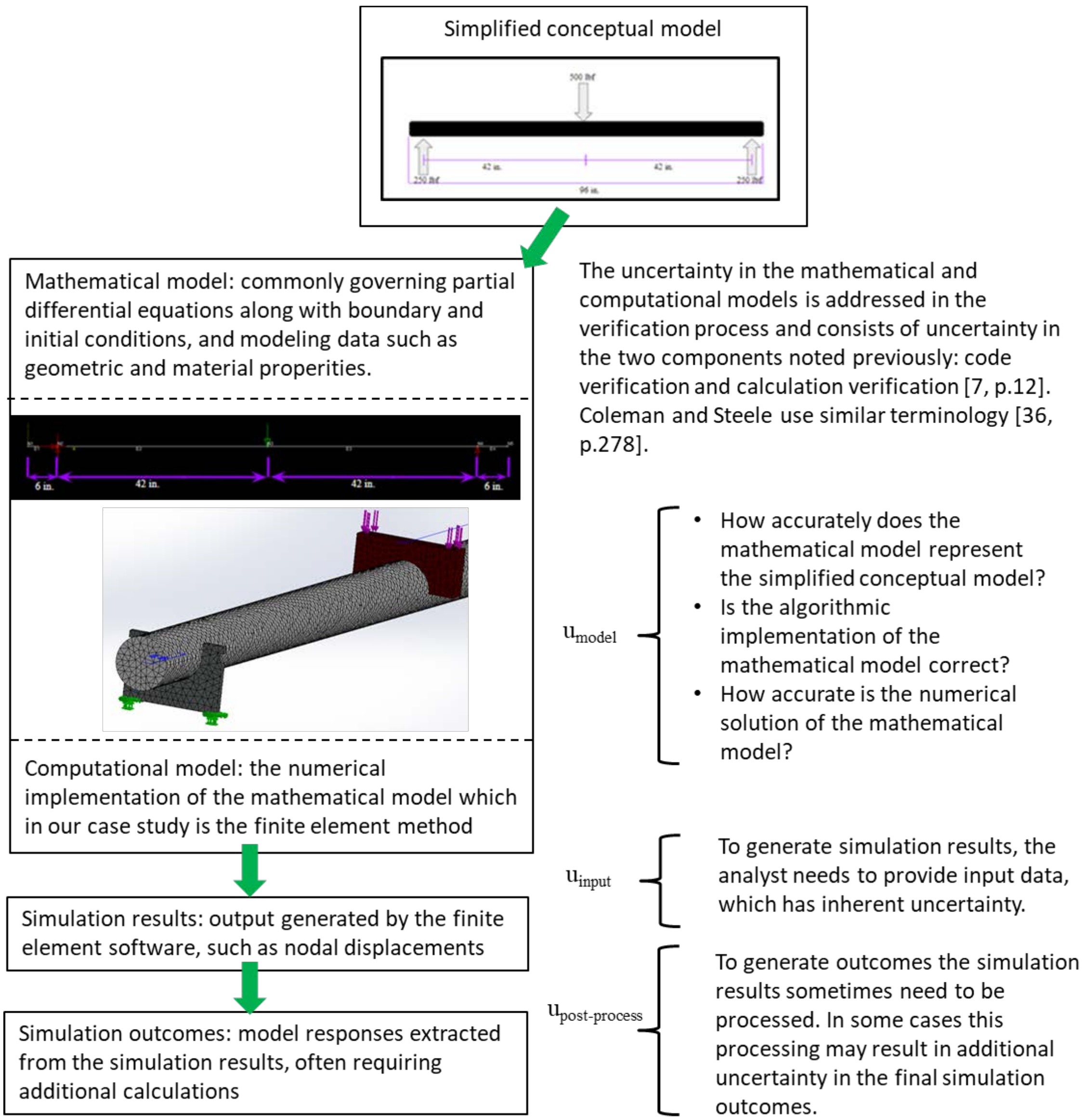


Figure 5. Sources of uncertainty in simulation outcomes

The only mainstream textbook we have found that addresses quantification of uncertainty in numerical modeling and simulation is Experimentation, Validation, and Uncertainty Analysis for Engineers [26]. It is worth noting that the first edition of this text published in 1989 did not have "Validation" in its title. Validation was added to the third edition in 2009 and the coverage continues to evolve as can be seen in the fourth edition (2017). The text provides a development of uncertainty analysis for physical experiments that is in line with the GUM, and applies similar concepts to computational simulations. In addition to this text, we found the material in [27], [28] and [29] helpful in furthering our understanding of VV&UQ in computational simulations. We were unable to find a standard for computational simulations published by an authoritative source comparable to those that exist for physical simulations.

As in the case of physical experimentation, we propose grouping sources of uncertainty into three components: u_{model}, u_{input} and $u_{post\text{-}process}$. Again, the general intent of the categorization is to promote contemplation and discussion of sources of uncertainty and to facilitate an equation for the quantification of the uncertainty in a computational outcome as being the sum of the uncertainties from all sources, as in Equation 7.

$$u_{outcome} = u_{model} + u_{input} + u_{post-process} \tag{7}$$

While in some respects u_{model} is akin to the term representing modeling uncertainty in a physical experiment, u_{model} in computational simulations also includes uncertainty in the creation and coding of algorithms representing the mathematical model, and uncertainty in the results of the calculations due to discretization error in the numerical solution of the equations. Uncertainty associated with the coding and calculations is typically addressed as part of the verification process. We see the UQ involved in the verification process as being beyond what can be accomplished by typical undergraduate engineering students and are looking for reasonable estimates that could be adopted and integrated into the uncertainty quantification process.

The uncertainty associated with input in Eqn. 7 is similar to $u_{measure}$ in Eqn. 3. In our case study, the beam span, cross-sectional diameter, and load represent input to the computational model, and carry associated uncertainty into the outcome. Additionally, material properties will have to be input, and will have uncertainty associated with them. In some simulations, input values may also be needed to characterize initial and boundary conditions, and their uncertainty would have to be accounted for.

As in the case of physical experiments, the results from a computational simulation may require post-processing to get the desired outcome(s). In this study for instance, the algorithms may not include the displacement specifically at the bottom of the cross section at midspan, but this value could be interpolated using displacements at nearby nodes. In this and similar cases, uncertainty in post-processing could be addressed as part of the code verification process and not contribute to the uncertainty in the outcome as a separate term. On the other hand, given the variety of output that most general purpose codes offer, there is often some significant uncertainty regarding whether or not students will correctly utilize the output when drawing conclusions.

With no standard to follow, we will quantify u_{input} using the analytical expression that was used in developing the DRE for the physical experiment, but in this case, written in terms of the unknown displacement at mid-span as shown in Equation 8. All of the terms on the right side of the equation represent input to the computational model, with the outcome of interest being the beam's centerline displacement. All of the assumptions mentioned previously in regard to Eqn. 2 apply.

$$\delta = \frac{4PL^3}{3D^4\pi E} \tag{8}$$

Choosing displacement as the outcome seems reasonable given that nodal displacement is a common basic output in finite element analysis. We again assume a Type B evaluation, and use our judgment to assign uncertainties to the parameters in Eqn. 8. In this case, we don't know, and can't directly measure E. Published data does not exist for a PE-PP blend so we used the rule of mixtures to estimate the elastic modulus. It turns out there is a large range of published values for the elastic modulus for polypropylene (PP). The assigned uncertainties are presented in Table 2.

Table 2: Input parameters and their associated uncertainties

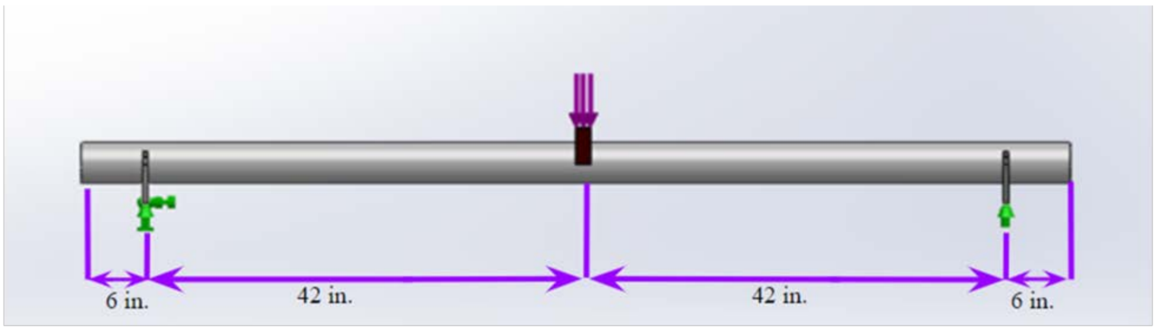
Input variable	Value	Uncertainty
P = load = x_1	500.0 lbf	± 2.5 lbf = w_{x_1}
L = beam span = x_2	84.0 in	± 0.5 in = w_{x_2}
D = beam diameter = x_3	4.00 in	± 0.25 in = w_{x_3}
E = elastic modulus = x_4	186000 psi	± 24000 psi = w_{x_4}

The result from Eqn. 8, and the uncertainty in that result determined with Eqn. 4, are 2.64 ± 0.74 in, representing a 28% uncertainty. While this large uncertainty was expected, the result is nonetheless informative. The level of uncertainty in a hand calculation for beam displacement often goes unaddressed in a mechanics of materials or structural analysis course.

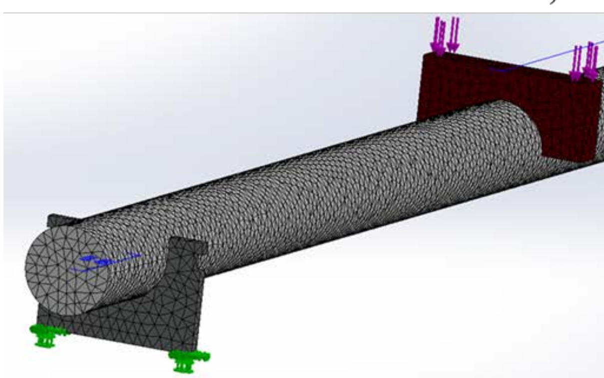
In Figures 1 and 4 in the ASME V&V-10-2006 guide [7], parallel mathematical and physical modeling processes are laid out, starting with the same "reality of interest" and progress towards the stage at which corresponding experimental and computational results can be compared. Figure 4 in V&V-10 [7, p.6] was in fact the inspiration for our Figures 3, 4 and 5. Our physical experiment wasn't intended to be a validation experiment, and given the lack of agreement between the displacement and strain data in the physical experiment, it would not qualify as a validation experiment. Instead our simulation work is currently more along the lines of a forensic investigation: What is the uncertainty associated with using simple beam theory to interpret our experimental results?

Our initial simulation efforts involved a model using Bernoulli-Euler beam elements in a code that was primarily built for simulating framed structures [30]. The results of these simulations verified that we were accurately using our DRE, and also gave some indication that our end supports were providing little restraint to rotation. We were also able to show that in addition to the material nonlinearity indicated by our test data, there could be some significant geometric nonlinear behavior in the beams if the end supports were constraining axial movement, which could well be the case if the beams were significantly distorting around the support cradles.

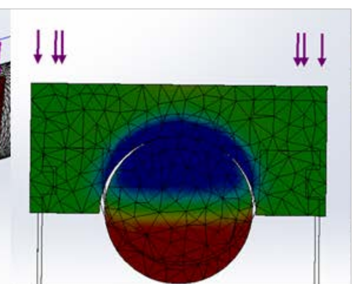
A 3-dimensional (3D) high fidelity model was subsequently created, and some preliminary simulations have been run using a general purpose code [31]. The model incorporated the experimentally found value of the elastic modulus (E), an assumed value of Poisson's ratio (ν) as well as the loading saddle and support cradles with assumed material properties for each. The computational model is shown in Figure 6. Tetrahedral elements are employed in the simulations, and a mesh sensitivity study will be conducted, but we will first need to decide on the appropriate parameters for the contact interfaces included in the model between the support cradles and the beam, and the loading saddle and the beam.



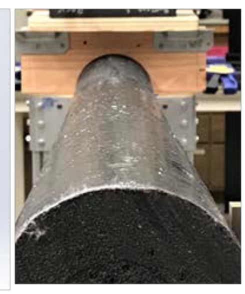
a) Plan view



b) Meshed model



c) Stresses on a cross section through the loading arch and beam



d) Loading arch interaction with a beam specimen

Figure 6. High fidelity FEA model

We anticipate being able to estimate the uncertainty in the parameters for the contact interfaces through a sensitivity study. We will add that estimate to our current estimate for u_{input}. In our judgment, $u_{post\text{-}process}$ for this study can be deemed relatively insignificant. Developing an estimate for u_{model} is a work in progress.

As previously noted, there is relatively little guidance in the literature regarding UQ for simulations that represent interpolations between, or extrapolations beyond validation points. This is an area of active and needed research [26], [27], [28], [29]. An especially interesting discussion of this issue, with a proposed approach to UQ when validation is not possible or practical, is presented in [32] where the authors address a broad spectrum of modeling activities where uncertainty quantification is needed to inform decision making.

Uncertainty Estimation for Analytical Models

Analytical models are likely more often encountered by undergraduate engineering students than physical models or computational models. An apt description of the FERM [21] is a collection of analytical models and associated input data. We used an analytical model as a data reduction equation in our UQ calculation for our physical modeling, and strongly suspect that a majority of physical experiments conducted in undergraduate engineering courses also use such equations presented in the FERM to derive quantities from experimental data to support student learning objectives.

Uncertainty quantification in association with the use of analytical models is easily accomplished with the K-M equation. Exercising the K-M equation in undergraduate engineering courses could obviously serve to improve students' designs of physical experiments, but could also serve to get students to think more critically when doing any calculation with an analytical model for the purpose of design.

Conclusions:

Our research indicates that relatively few engineering undergraduates are being exposed to uncertainty quantification beyond using standard rules for appropriate numbers of significant figures in a reported result. While students are quite often required to address "does the answer make sense" as a final step in a calculation, they are not generally required to include UQ in this discussion. Reporting a result for E for a polymer blend as being 186 ksi sounds authoritative, but when reported with its associated uncertainty as 186 ± 50 ksi, subsequent design decisions would likely be different.

Very few texts that we examined cover UQ, and this may be part of the reason for the lack of coverage of UQ in many engineering courses. Regardless, we suspect that some sophomores, and most juniors and seniors have the background to quickly pick up and use the versions of the K-M equations presented in the FERM [21]. Given the single-sample nature of the application of many physical and analytical models in undergraduate studies, the K-M equation in the "Instrumentation, Measurement, and Control" section of [21] may be the more useful option.

The equations we have proposed for use in UQ in physical (Eqn. 3) and computational (Eqn. 7) simulations are a work-in-progress. We believe that maximizing commonality in UQ in physical and computational situations will enhance student learning. More thought, and input from other educators, is needed. Of special interest is promoting use of the terms verification and validation in physical experimentation studies that will prepare students to understand those terms in the computational realm.

UQ can and should play a significant role in the development and conduct of physical experiments by students, but can also be used to help students develop their engineering judgment. When analyzing and interpreting experimental data, and drawing conclusions from that data, students should be reflecting on the level of uncertainty inherent in the data. Similarly, when analyzing and interpreting output from computational simulations, students should be cognizant of the inherent uncertainty in computer-generated results, and their "informed judgments" [12, p. 5] should be conditioned by that uncertainty.

Physical experiments and computational simulations are being embedded more and more into the undergraduate engineering curriculum. We see UQ as an urgent need for the engineering community and particularly engineering students. Our hope is to design curriculum enhancements that piggyback on already existing content in the form of short exercises that complement homework and lab activities by bringing in the layer of uncertainty quantification. We see use of UQ in conjunction with the use of analytical models throughout engineering curriculum as a convenient way to build facility with UQ.

In addition to students who are more effective users of engineering models and simulations, we expect engineers trained in uncertainty quantification will be more critical consumers of results from modeling in other fields. They will routinely approach simulation results with the question “Are you sure about that?”

References

1. Gallagher, R.H., Finite Element Analysis Fundamentals, p. xi,, Prentice-Hall, 1975
2. Fish, J., and Belytschko, T, A First Course in Finite Elements, p. 3, Wiley, 2007
3. Froyd, J.E., Wankat, P.C., and Smith, K.A., “Five Major Shifts in 100 Years of Engineering Education,” pps. 1353-1354, Proceedings of the IEEE, Vol. 100, May, 2012
4. Magana, A.J., “Modeling and Simulation in Engineering Education: A Learning Progression,” Journal of Professional Issues in Engineering Education and Practice, ASCE, 2017, DOI: 10.10161/(ASCE)EI.1943-5541.0000338
5. Lenox, T.A., Ressler, S.J., O’Neill, R. J., and Conley, C.H., “Computers in the Integrated Civil Engineering Curriculum: A Time of Transition,” Proceedings of the 1997 Annual Conference of the American Society for Engineering Education, June, 1997
6. Prantil, V.C., Papadopoulos, C., and Gessler, P.D., Lying by Approximation: The Truth about Finite Element Analysis, Morgan and Claypool Publishers, 2013, DOI: 10.2200/S00503ED1V01Y201305ENG023
7. American Society of Mechanical Engineers (ASME), Performance Test Code Committee 60 (PTC 60), Guide to Verification and Validation in Computational Solid Mechanics, (ASME V&V 10-2006), 2006
8. Ben Gammell, for Your Public Media, “Almost a Tragedy: The Collapse of the Hartford Civic Center,” CONNECTICUTHISTORY.ORG, https://connecticuthistory.org/almost-a-tragedy-the-collapse-of-the-hartford-civic-center/ accessed 11 January, 2021
9. Levy, M and Salvadori, M, Why Buildings Fall Down: How Structures Fail, W.W. Norton, 2002
10. Feld, J., and Carper, K.L., Construction Failure, 2nd Ed., Wiley Interscience, 1997
11. National Transportation Safety Board (NTSB), Pedestrian Bridge Collapse Over SW 8th Street, Miami, FL, Highway Accident Report HAR1902, NTSB, adopted October, 2019
12. ABET, Criteria for Accrediting Engineering Programs, https://www.abet.org/wp-content/uploads/2020/09/EAC-Criteria-2020-2021.pdf
13. Morrison, M., “Models, measurement and computer simulation: the changing face of experimentation,” Philosophical Studies, Springer, 2009, DOI 10.1007/s11098-008-9317-y
14. Kelton, W.D., and Barton, R.R., “Experimental Design for Simulation,” Proceedings of the 2003 Winter Simulation Conference, pps. 59-64, IEEE

15. American Institute of Aeronautics and Astronautics (AIAA), Guide for Verification and Validation of Computational Fluid Dynamics Simulations, approved by the CFD Committee on Standards 1998, reaffirmed 2002
16. National Aeronautics and Space Administration (NASA), Standard for Models and Simulations, NASA-STD-7009A, 2016
17. National Aeronautics and Space Administration (NASA), NASA Handbook for Models and Simulations: An Implementation Guide for NASA-STD-7009, 2019
18. Davis, H.E., Troxell, G.E., and Wistocil, C.T., The Testing and Inspection of Engineering Materials, Chapter X, McGraw-Hill, 1941
19. Baird, D.C., Experimentation: An Introduction to Measurement Theory and Experiment Design, Prentice-Hall, 1962
20. Taylor, J.R., An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements, 2nd Ed., University Science Books, 1997
21. NCEES, FE Reference Handbook 10.0.1, 2020
22. Kline, S.J., and McClintock, F.A., "Describing Uncertainties in Single-Sample Experiments," Mechanical Engineering, ASME, pps. 3-8, January, 1953
23. Joint Committee for Guides in Metrology (JCGM), International vocabulary of metrology – Basic and general concepts and associated terms (VIM), 3rd Ed., JCGM 200:2012, downloaded from https://www.bipm.org/en/publications/guides/ November, 2020
24. Joint Committee for Guides in Metrology (JCGM), Evaluation of measurement data – Guide to the expression of uncertainty in measurement (GUM 1995 with minor corrections),JCGM 100:2008, downloaded from https://www.bipm.org/en/publications/guides/ November, 2020
25. Carter, A.R., "Evolution of the Significant Figure Rules," The Physics Teacher, American Association of Physics Teachers, 2013, DOI 10.1119/1.4818368
26. Coleman, H.W. and Steele, W.G., Experimentation, Validation, and Uncertainty Analysis for Engineers, 4th Ed., Wiley, 2018
27. Schwer, L.E., "An Overview of the PTC 60 / V&V 10 Guide for Verification and Validation in Computational Solid Mechanics," Engineering with Computers, 23(4):245-252, DOI: 10.1007/s00366-007-0072-z, 2007
28. American Society of Mechanical Engineers (ASME), V&V-10 Committee, An Illustration of the Concepts of Verification and Validation in Computational Solid Mechanics, (ASME V&V 10.1-2012), 2012
29. Roy, C.J. and Oberkampf, W.L., "A comprehensive framework for verification, validation, and uncertainty quantification in scientific computing," Computational Methods in Applied Mechanics and Engineering, 200 (2011) 2131-2144, Elsevier
30. W. McGuire and R. Ziemian, [MASTAN2], [v.3.5], Available: mastan2.com , Accessed: Jan. 2021
31. SOLIDWORKS, Dassault Systemes, [SolidWorks Modeling, SolidWorks Simulation], [2018-19], Available: https://www.solidworks.com/
32. Christie, M.A., Glimm, J., Grove, J.W., Higdon, D.M., Sharp, D.H., and Wood-Schultz, M.M., "Error Analysis and Simulations of Complex Phenomena," Los Alamos Science, Los Alamos National Laboratory, Number 29, 2005