An International Study of Foucault's Pendulum

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Abstract

Léon Foucault proved the rotation of the Earth with Foucault's Pendulum experiment using a pendulum with 67 meters in length. A Foucault's Pendulum refers to a heavy mass swinging about a relatively high pivot point, where the inertial plane of the pendulum's swing rotates over time. This rotation of the plane is called precession. Due to the Earth's rotation, the precession is clockwise in the northern hemisphere and counterclockwise in the southern hemisphere. There is no precession at the equator and the pendulum keeps swinging in the same direction. Our small-scale Foucault's Pendulum experiments were carried out in two different countries as an independent study by a sophomore engineering student. Project Based Learning (PBL) was applied in this study. Since Foucault's experiment is in such a big scale, logistically it is hard to demonstrate it at schools. The aim of this study was to create a small-scale pendulum so other students and teachers around the world can easily build one to prove the Earth's rotation. Using a small-scale setup, about 3 meters in length, we studied the effects of the Earth's rotation on the pendulum's precession with identical materials (i.e., object's mass and size, length of the string, starting amplitude, etc.). This experiment was conducted at the Kinsley Engineering Center of York College of Pennsylvania, York, Pennsylvania, United States and in a local residence located in San Nicolas de los Arroyos, Buenos Aires, Argentina, where the two cities have similar but opposite latitudes which should yield precessions of similar magnitudes but opposite directions. Several factors were investigated for the small-scale Foucault's Pendulum such as the influence of air resistance, the effect of releasing the object for minimum perturbation, and the elliptical path of the object. The data gathered in the two countries were compared. In this study, we are not only presenting the educational objectives of the independent study but also the design process of the small-scale pendulum which can be reproduced anywhere else in the world with a low budget.

Introduction

Léon Foucault first started publicly conducting his famous experiment in 1851. His demonstration (now dubbed the Foucault Pendulum) has since been recreated in museums and universities worldwide. He suspended a 28-kg iron coated lead ball from a height of 67-m from the dome of the Panthéon, Paris, France [1]. He demonstrated how the precession was predictable according to the latitude of the location of the pendulum. Although impressive, this setup is not necessarily easy to reproduce at this scale.

To understand the theory behind a pendulum's precession, the student was directed to various papers [1-6]. Although the math behind is heavy for a sophomore student, the student focused on the papers that studied the Foucault Pendulum experiment through geometrical

models [3,4]. They served to understand how an inertial plane moves around a spherical object and how changing the suspension point from the axis of rotation to the equator (in the case of the Earth) lengthens the period of rotation by the following formula:

$$T = \frac{T_0}{sin\,(\theta)}$$
Equation 1 [3,5]

where T is the period of precession in hours, T_0 is the duration of a sidereal day (i.e., one 360° rotation of the earth, instead of relative to the sun) which equals to 23.943 hours, and θ is the latitude of the pendulum's location which is measured from the equator towards the axis of rotation (the Earth's poles). Utilizing this formula, the periods of precession of the selected locations in USA and Argentina can be calculated. Also, the angle of precession after 1 hour was calculated as this is related to how the experiments are conducted.

United States of America: Kinsley Engineering Center, York, Pennsylvania. Latitude: 39.94° N
Argentina: Local Residence, San Nicolás de los Arroyos, Buenos Aires. Latitude: 33.36° S

For brevity, the locations will be referred to as "USA" and "Argentina" for the remainder of this paper. Table 1 presents the calculated full rotation precession periods (T) and precession angles after 1 hour (θ_{1h}).

Table 1: Precession periods and angles

Location	Latitude (θ)	Direction	T	θ_{1h}
USA	39.94° N	Clockwise (CW)	37.297 hours	9.65°
Argentina	33.36° S	Counter clockwise (CCW)	43.541 hours	8.27°

As calculated, the precessions were similar in magnitude with opposite precession directions. The experiment was designed for about an hour trial and precession was compared to that of expected.

*PBL methodology integration to the Independent Study***:**

This project was centered on the open-ended question: Can the Foucault's Pendulum be accurately recreated in a classroom environment with a low budget? To conduct the activities needed to answer this question, the student requires not only the knowledge of physics, but also research skills, geometry, modeling, etc. The Foucault's Pendulum experiment may be discussed briefly in classrooms but often not in great depth.

From classroom experience, pendulum problems are often simplified by many assumptions that reduce their complexity to allow for a solution obtainable in the span of a

lecture. However, their real-life counterparts are influenced by many factors such as air drag, line elasticity, how the suspension point is compared to a fixed pivot point, whether or not the suspension point moves, the center of gravity of the bob, air movement, etc. Some of these factors were not anticipated to play dominant roles in the results in the beginning of the study but later they were rather encountered as challenges and their external influence over the pendulum had to be mitigated. The height of the pendulum can also be problematic. A great height would best resemble the original experiment conducted by Léon Foucault but logistically it is hard to create it and it would not be possible to reproduce in many schools.

The project is open to iterative design, where the way the pendulum is set up is researched, designed, implemented, tested, and then checked for errors, redesigned and so on. This iterative design is supervised by the professor. The student was guided in background research, then, the design of the setup was left mostly to the student with further guidance in the setup design when necessary. The student also learned about how to keep a scientific journal and at the end of the project the journal was a part of his grade for the independent study. Since the start of the project, we aimed for publishing our work and the student gained experience with writing a scientific paper as well.

Learning Objectives of The Independent Study:

Based on the structure of PBL, the following guidelines and expectations were put in place to be presented as an Independent Study. As mentioned earlier, the student who worked on this project was a sophomore and being the first independent study conducted by him (earlier than usual), the following essential scientific practices were set:
- Understanding the research process.
- Getting familiar with a scientific research including but not limited to literature search, designing and executing the experiment accurately.
- Recording data and results on a research journal.
- Gathering data accurately, in SI units. Calculating error.
- Designing an experiment that can be reproduced easily.
- Overcoming challenges as the research progresses.
- Writing a scientific paper.
- Improving time management.

The student is pursuing his electrical engineering education at York College of Pennsylvania, USA and has connections in Argentina. After completing preliminary research in USA, he repeated the same experiment in Argentina and communicated with Argentina for the rest of the experiment and data taking.

Design Process

Experimental Setup:

In order to be able to measure the inertial plane of the pendulum, a measuring device was created by the student. Named the "Planar Precession Compass" (PPC, or Compass in this paper), it functions by forming a "plane" perpendicular to the floor which can be aligned with the pendulum's suspension line while in motion. The initial concept in hand sketches can be seen in Figure 1.
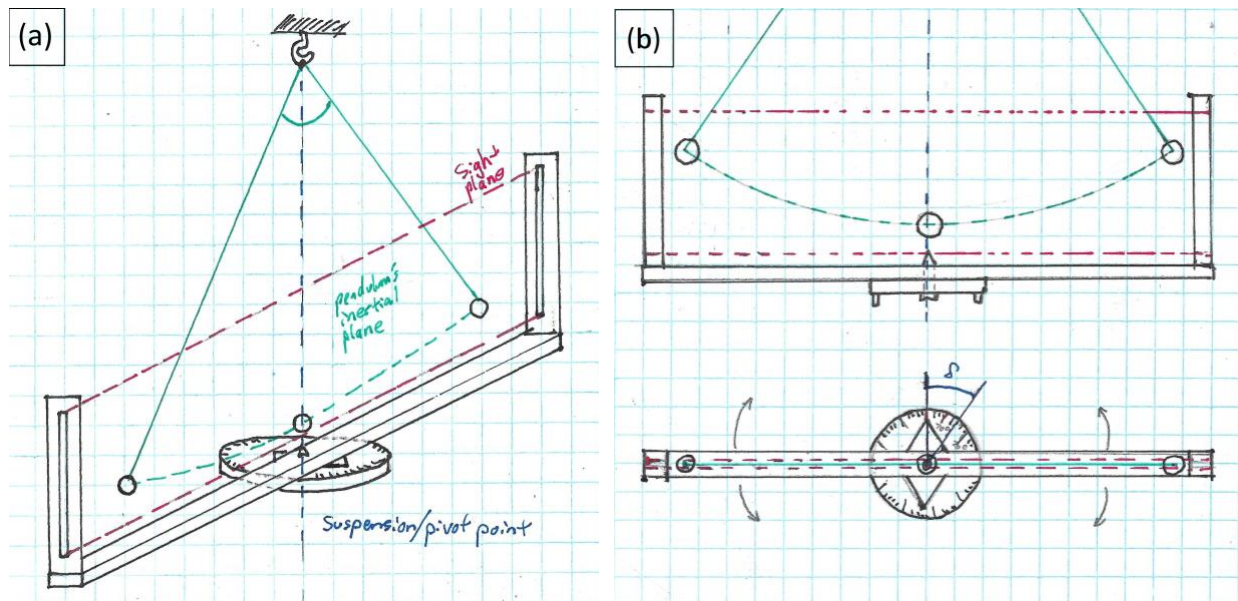

Figure 1. Initial hand sketches of Planar Precession Compass from the student's journal.
(a) Diagonal, (b) side and top views.

Figure 1(a) illustrates two vertical slits on two vertical bars creating a visual plane through which the pendulum line can be seen. The PPC then rotates along the axis of the suspension point of the pendulum until it is aligned with the pendulum's inertial plane. The pendulum and the compass are aligned when the suspension line only moves forwards and backwards when looking through the vertical slits.

This concept was 3D modeled in SolidWorks by the student and all parts, except the mounting bar, were 3D printed. This bar was machined from an aluminum bar as 3D printing such a large part would be impractical. Figures 2(a) and 2(b) show the 3D model and the finished assembly, respectively. The base plate features a circle graded every 1°, printed on a piece of paper that is centered to the plate. This allows the user to measure the rotation of the compass as marked by the base dial, seen in red in Figure 2. This rotation of the compass equals the

precession of the pendulum, marked in blue in Figure 1(b) as δ, but in this paper θ will represent the precession of the pendulum.
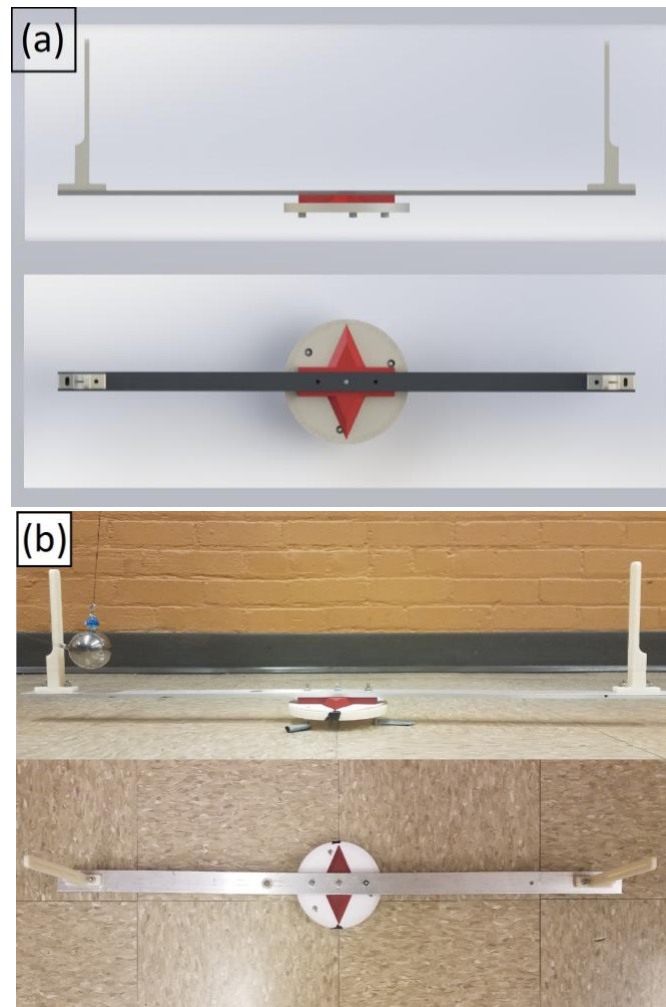

Figure 2: (a) SolidWorks model of the PPC (b) finished assembly of the PPC.

In order to function properly, the rotation axis of the PPC must coincide with the suspension line of a resting pendulum, and therefore the base plate must be leveled. Three screws were used to level the baseplate, as can be seen in Figure 2(a). However, this calibration resulted to be tedious and difficult to implement. Furthermore, since the user must visually see the pendulum swing through the vertical slits, they must either swing the pendulum above a table, which can be bumped and moved thus ruining the experiment, or the PPC must be placed on the floor, which while more stable, is inconvenient to the user since they must lay on the floor to look through the slits. These two problems were solved by replacing the way the PPC creates its alignment "plane". Instead of a visual plane, an auto-leveling laser level was used to simulate a vertical plane as seen in Figure 3.
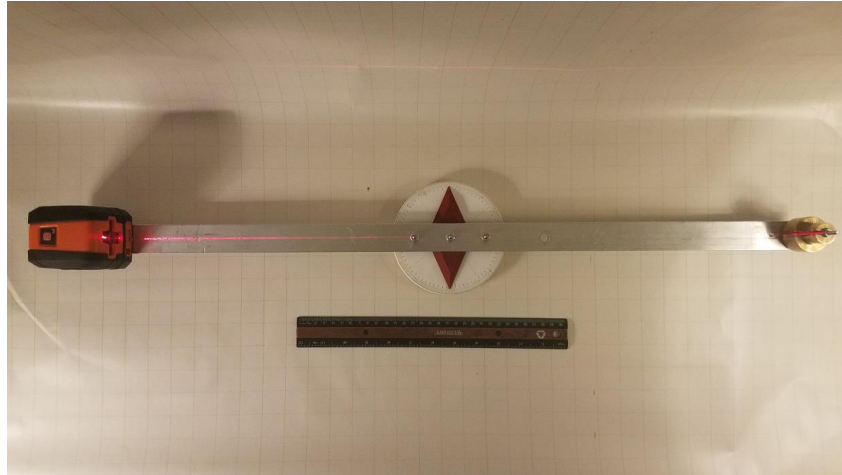
Figure 3: Final version of the PPC

This auto-leveling feature ensures that the generated plane is vertical even if the base plate and mounting bar are off level. In the experiments, both the vertical bar version and laser level versions of the PPC were used. Now, having a device that can measure precession of the pendulum, the experimental setups were created.

Initial Set-up:

The Foucault Pendulum can be simplified into three components: mass, suspension line and suspension point. An initial selection of these components was made, taking into consideration their availability. To consider easy reproduction, readily available and inexpensive materials were used. Mass: Initially, two dense steel balls with an average mass of 490.5-g and a diameter of 4.5-cm were used. Figure 4 shows the variety of balls used for this project and the steel ball used for the initial setup is shown on the right.

Figure 4: Pool ball (left), bowling ball (center), steel ball (right)

Suspension line: In an ideal problem, the suspension line of a pendulum would have no mass, would not cause drag and would not deform (stretch). To best approximate this for our real-world experiment, we considered the following choices of suspension lines: Initially, a steel wire was considered, but was ruled out due to its relatively high density. A braided plastic fishing line was considered, which is said to have low stretch and is very light. This was tested using a tensile tester and was calculated to stretch 2.7 mm if suspending a 0.5 kg ball from 3.0 m height. This is a 0.9% stretch, which we considered as an acceptable error. Figure 5 shows the stretch test results. Suspension point: For the first trial, a simple C-clamp with perpendicular bars was used, as can be seen in Appendix B.
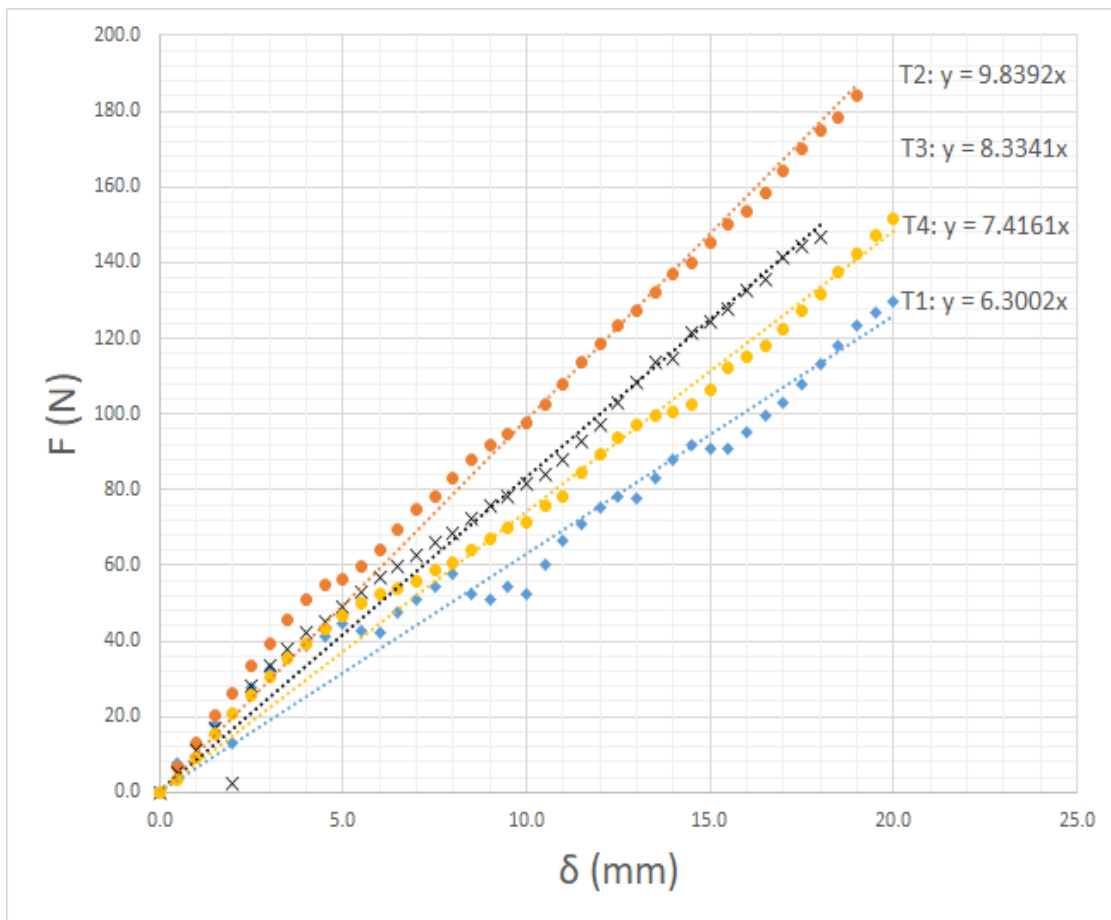

Figure 5: Stretch test results for a fishing line used in the pendulum setup. Average of four trials are used to measure the amount of stretch.

Having all these components and the first iteration of the PPC (with the vertical bars), a first setup was established. The C-clamp was clamped to the I-beam of a ceiling in Kinsley Engineering Center. From this point, one of the metal balls were suspended with braided fishing line. While motionless, the PPC is placed under the pendulum and "centered". The ball is then pulled back laterally 35-cm (as close to the vertical beams of the PPC as possible). This is done by a cotton string (retraction line) which is to be burned to initialize a "Trial". This complete

setup can be seen in Figure 2(b). Burning the retraction line ensures that no lateral energy is introduced into the system, as would happen if the line were cut with a scissor, for example. Furthermore, each trial was given an initial "wait time", a time between when the mass is pulled back to when the mass is launched. Even after holding the mass in place, some energy is always present, as can be seen by the mass moving slightly. This time is to allow for said energy to attenuate before starting a trial.

Next, the student suspected that the air resistance and air movement could influence the motion of a pendulum. This doubt emerged when, while conducting a trial, the retraction line was burnt. It was noticed that the flame of the lighter flickered and "dragged" in one direction, showing that there was air flow in the room. Several points were tested in the room and different directions of air flow were seen. This may have influenced the precession of the compass, as seen in the initial trials section of this paper. Therefore, different locations and methods of stopping airflow were tested. The experiment set-up was moved to two other locations where the air ducts of the ceiling were taped over. Also, a "curtain" was set up at the base of the pendulum, to further reduce the air flow around the pendulum.

Another factor that was overlooked until later in the experiment was the suspension point. Initially, the fishing line was looped around the C-clamp's arm. It was tied tight using the provided fishing knot from the manufacturer (Appendix F), but nevertheless always maintained some slack in between the knot head and the arm. This suspension point was further from ideal than desired. Also, the C-clamp itself could potentially bend laterally, further influencing the precession. To mitigate this, a new suspension block was created, as seen in Appendix C. Here, the block itself is clamped firmly to a ceiling I-beam. The line was held in place within a cut off eraser end, which was then inserted into a tight-fitting hole and then glued in place. Now, the suspension point resembles an actual point in space much more closely than the previous knot/clamp arrangement which made the suspension point a fixed pivot point as desired. Also, the mass was changed from the steel ball to a bowling ball with a 6600-g mass (Figure 4, center). The larger mass (compared to the steel balls) allows for the effect of air drag to have less influence over the pendulum's motion.

Results

Figure 6 Trial Set A shows the first trials that served to set the scope of how the following trials would be conducted. After about 35-40 minutes, the pendulum's amplitude (distance from the center to the peak of its swing, as seen from above) was about 4-cm, in between the screws that hold the PPC dial in place (see Figure 3). This amplitude was considered to be too small to measure precession accurately, therefore a time of 30-min was chosen as the end of the trial. First, the direction of the pendulum's precession oscillated back and forth, with the pendulum also taking an elliptical path. It initially swung in a "straight" plane while it started

to precess CCW. As the pendulum precessed further, the swing took an elliptical path in CW direction. At about the 20-minute mark, the pendulum then stopped precessing and took a straight path once again. Finally, a CW precession was observed with a CCW elliptical path until the end of the trial. Note that missing trial numbers were botched trials caused by disturbing the pendulum and were taken out. Next, the effect of the direction of initial launch of the pendulum was investigated as seen in Figure 6 Trial Set C. Trials 1, 2 and 3 were pointed in directions North/South, East/West, North/South, respectively. This change in initial direction also changed the precession direction. The magnitude of the precession varied with the precession direction as well. In this case, the reversal of precession did not occur. In Figure 6 Trial Set D, trials 1, 2 and 3 were pointed in directions East/West, North/South, East/West, respectively. The same change in precession direction and magnitude as in Figure 7 is observed here, with the lack of precession reversal. Figure 6 Trial Set E shows the results obtained in Argentina. As in Figure 6, using similar parameters, the pendulum precesses in one direction initially, and reverses around approximately 20 minutes. The height of the pendulum was approximately 1 m shorter, but still observed a similar precession as in the USA.

Table 2: 3-meter scale setup parameters with steel ball

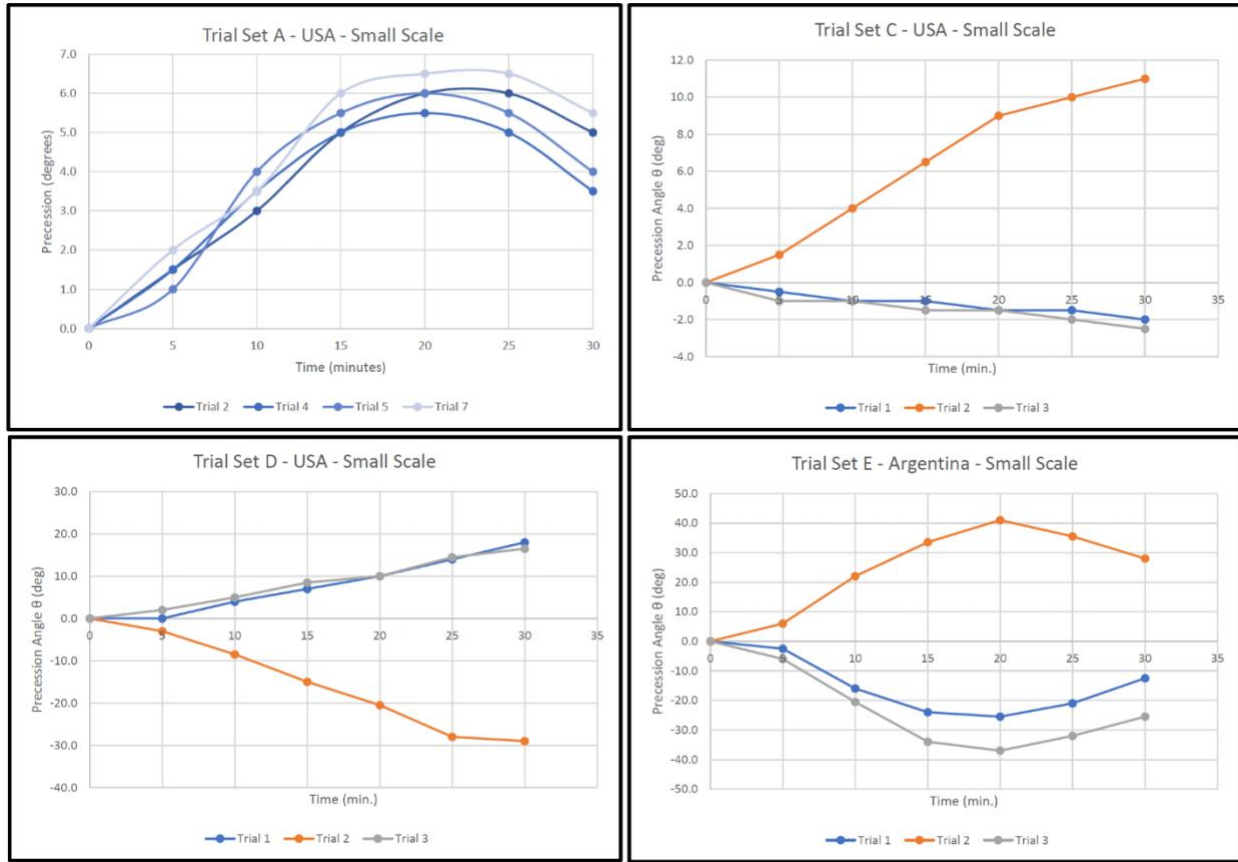| Trial Set	Trial Location	Mass	Pendulum Length	Initial Wait Time	Trial Duration	Suspension Point	Alignment Plane
A	USA	0.4904 kg	361.3 cm	5 min	30 min	C-clamp	Vertical Bars
C	USA	0.4904 kg	354.7 cm	10 min	30 min	C-clamp	Laser Level
D	USA	0.4904 kg	361.3 cm	10 min	30 min	C-clamp	Laser Level
E	Argentina	0.4904 kg	273.0 cm	10 min	30 min	C-clamp	Laser Level

Figure 6: Precession results of a steel ball on 3-m pendulum. Table 2 data used for these trials. The missing Trial Set B was a botched set.

The next step was to try with a much heavier ball. The pendulum conducted with the bowling ball maintained a greater amplitude throughout the trials for which the trial time was easily extended to 60 minutes as seen in Figure 7. Even after said time, the amplitude did not approach the mounting screws of the PPC but was stopped at 60 minutes since this was considered sufficient data. Furthermore, the initial wait time was extended to 20 minutes, to allow for energy attenuation. Trial Set F had an initial direction of North/South. When placing the mass on the line, it was seen to stretch about 3.1 cm more than the lighter steel ball as expected. The next step was changing the direction of launch to compare the effects for the heavier ball. Trial Set G had an initial direction of East/West as seen on Figure 7. This change in initial setup direction from Trial Set F to Trial Set G did not change the precession direction unlike it did for the steel ball Trial Sets C, D and E.

Table 3: 3-meter scale setup parameters with bowling ball

Trial Set	Trial Location	Mass	Pendulum Length	Initial Wait Time	Trial Duration	Suspension Point	Alignment Plane
F and G	USA	6.60 Kg	355.4 cm	20 min	60 min	Suspension block	Laser Level
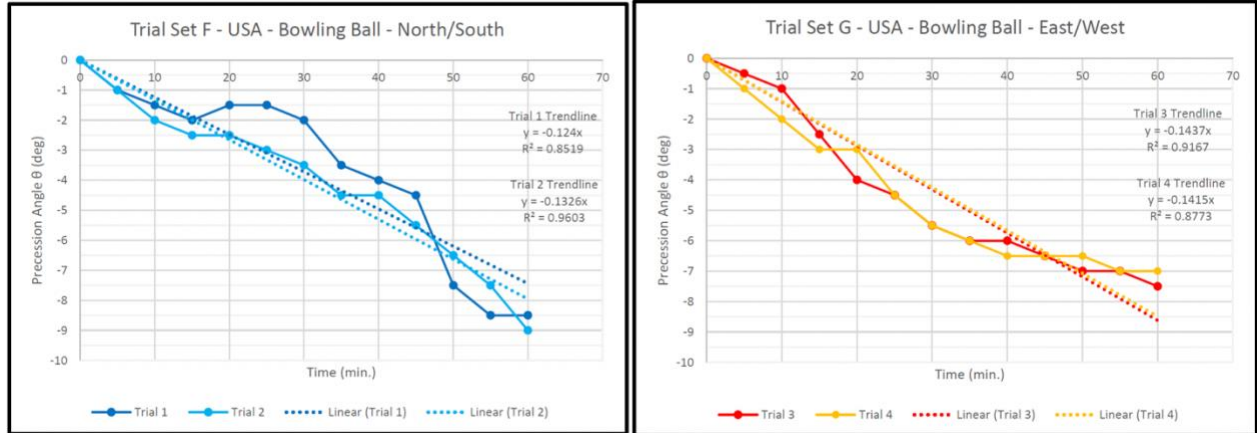
Figure 7: Precession results of a bowling ball on 3-m pendulum. Initial launch directions are North/South for Trial Set F and East/West for Trial Set G. Table 3 data used for these trials.

Since a steel ball was used, magnetic effects were also considered by the student. To understand the effect of magnetism, the steel ball was replaced with a non-magnetic pool ball. The precession direction did not change, and it followed a more linear displacement as seen in Figure 8. Also, the amplitude of the pendulum's swing was considerably smaller than the steel ball after 30-min (approaching the screws located at 4-cm from the center of the PPC) due to the light mass of the pool ball. Furthermore, even after 10-min of initial wait time, the pendulum could still be seen slightly swinging, which was not the case with the steel ball.

Table 4: 3-meter scale setup with non-magnetic pool ball.

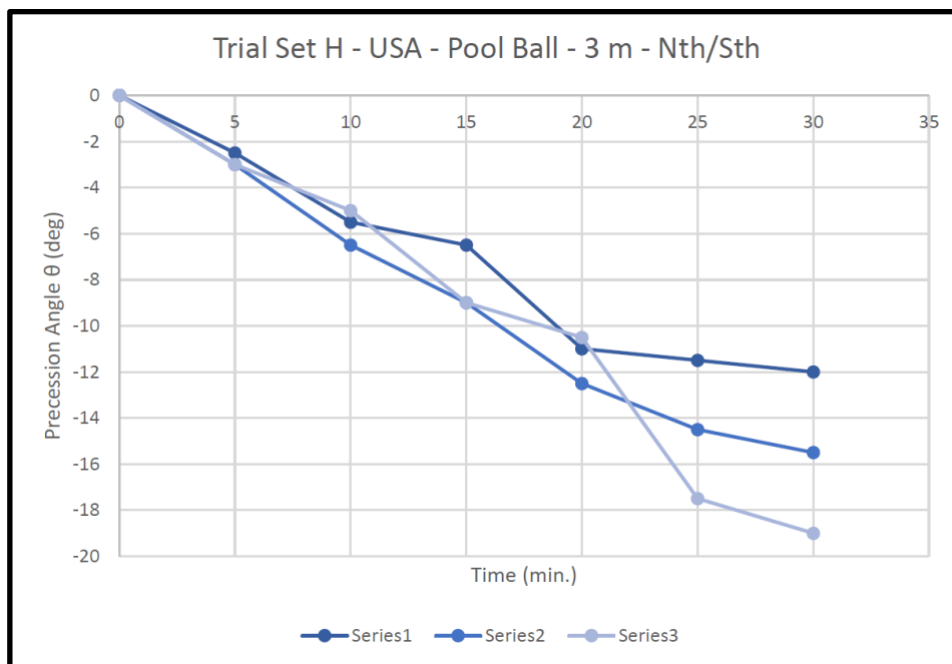| Trial Set	Trial Location	Mass	Pendulum Length	Initial Wait Time	Trial Duration	Suspension Point	Alignment Plane
H	USA	0.1820 kg	358.6 cm	10 min	30 min	Suspension block	Laser Level

Figure 8: Precession results of a non-magnetic ball on 3-m pendulum. Table 4 data used for these trials.

Discussion

Comparison between Trial Sets A, C, D and E as seen in Figure 6 (steel ball, USA and Argentina) suggested that changing the pendulum's initial direction may not only change the precession direction but also cause the precession to reverse. This, however, was not expected and it should have been an error. After considering several factors that affect the precession, the suspension point was investigated carefully. Although the suspension line was wrapped around the C-clamp arm and a special knot was used to make it a fixed point (Appendix F), a slight movement in this suspension point changed the precession. The student overcame this challenge by designing a new suspension apparatus and, in addition to previous precautions, the line was made to pass through a hole at the suspension point as explained earlier (Appendix C). Using a piece of rubber of the same size as the small hole of the apparatus made the line tight. Therefore the suspension point was fixed. This was first used in Trial Set F in Figure 7 with the bowling ball.

All trials, except the one conducted with the bowling ball as the mass, did not follow the predicted precession magnitude. To recall, pendulums in the USA should precess 9.65° in one hour and 4.83° in half of one hour as calculated in Table 1. Observing trial 2 from Figure 6 Trial Set C, it shows the pendulum precessing 11.0° in 0.5 hours. In one sidereal day, the pendulum would precess 526.7°. This is more than 360°, and it is not realistic unless external energy is added to the system. This large precession characteristic did not change when utilizing a non-magnetic mass, in Figure 8 Trial Set H (pool ball). The precession in this trial did not reverse,

however both the mass material and the suspension point were changed between these setups, therefore it could have been either change that stopped the reversal. Note also that the elliptical motion of the small mass pendulums, along with larger than expected precession mentioned previously, is characterized by Aczel and Romer [1].

Regarding the trials in Argentina, the results did not match those expected, since the reversal of the precession was observed similarly to the ones conducted in the USA. Their common point is that these trials used the C-clamp as its suspension point, which may explain this reversal. However, precession was still witnessed, showing that this effect can indeed be observed on opposite hemispheres.

Finally, the bowling ball trials (Figures 10 and 11) showed the most promising resemblance to the original Foucault's Pendulum. Changing the initial direction (that is, whether the pendulum initially pointed North/South or East/West) did not change the precession direction. In fact, it always precessed in the expected CW direction. The data obtained appeared linear and a linear fit trendline revealed R^2 values in the range of 0.85-0.95. Utilizing these trendlines, average precession angle after one hour was calculated to be 8.09°. This is a 16.18% error from the 9.65° calculated from Formula 1. Considering that this experiment was designed with a 3-m height instead of the original 67-m of Foucault's original experiment, this error is still acceptable and proves the point that a Foucault's Pendulum experiment can indeed be recreated on a small scale. Therefore, we suggest that the finalized apparatus with 3-m length can be used with a heavy ball like a bowling ball to study or illustrate the Earth's rotation.

Conclusion and Future Research

The paper focused on a design project to recreate the Foucault's Pendulum in a much smaller scale so it can be widely used around the world for education purposes. PBL methodology was used and the professor acted as a facilitator rather than providing all materials and directing the student at every step. Working on an independent study using PBL methodology has sharpened the student in research. He gained many skills while overcoming challenges along the way. His note taking skills have been greatly improved and he is currently keeping journals for other project-based courses as well.

Since the required suspension point at a height of 3-m can be easily found and the required materials are readily available (with the rise of availability of 3D printers), this experiment can be reproduced easily by high school and college students. Furthermore, an instructor can demonstrate the Earth's rotation using this small-scale setup to their students in real time.

As future research, more data will be collected in Argentina, especially one with a bowling ball in the same small scale. Many major cities in Europe share close latitudes to York, PA, USA, such as Madrid, Spain; Naples, Italy; and Istanbul, Turkey and will have similar precession angles and the experiment can be repeated and compared. If preferred, having been able to design the setup and mitigate the errors iteratively, people who would want to recreate this experiment would be able to start out from a working setup without having to spend time redesigning it, allowing for far better time efficiency in gathering data. Creating a larger pendulum at 10-m or more would make the experiment more accurate and closer to the original Foucault's Pendulum experiment if it were logistically possible. A journal was started early into the experiment, holding many designs and test results. However, keeping a more linear and permanent record of the experiment would have also been very useful.

Acknowledgements

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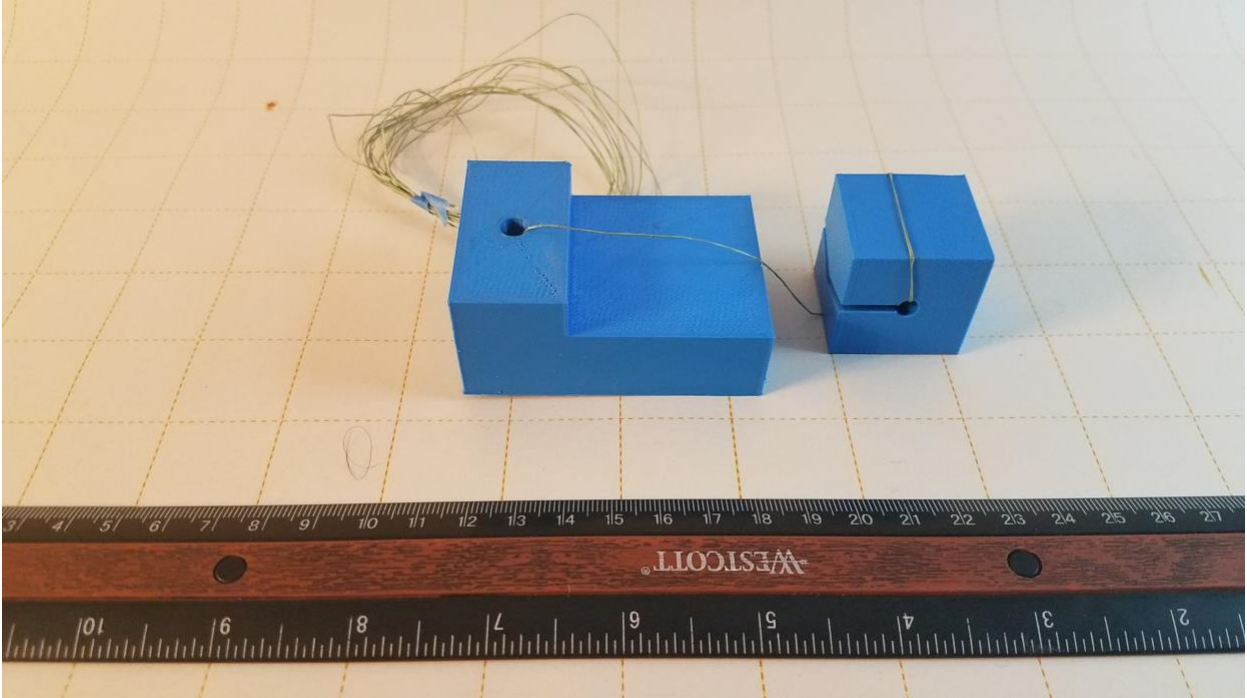
Appendixes
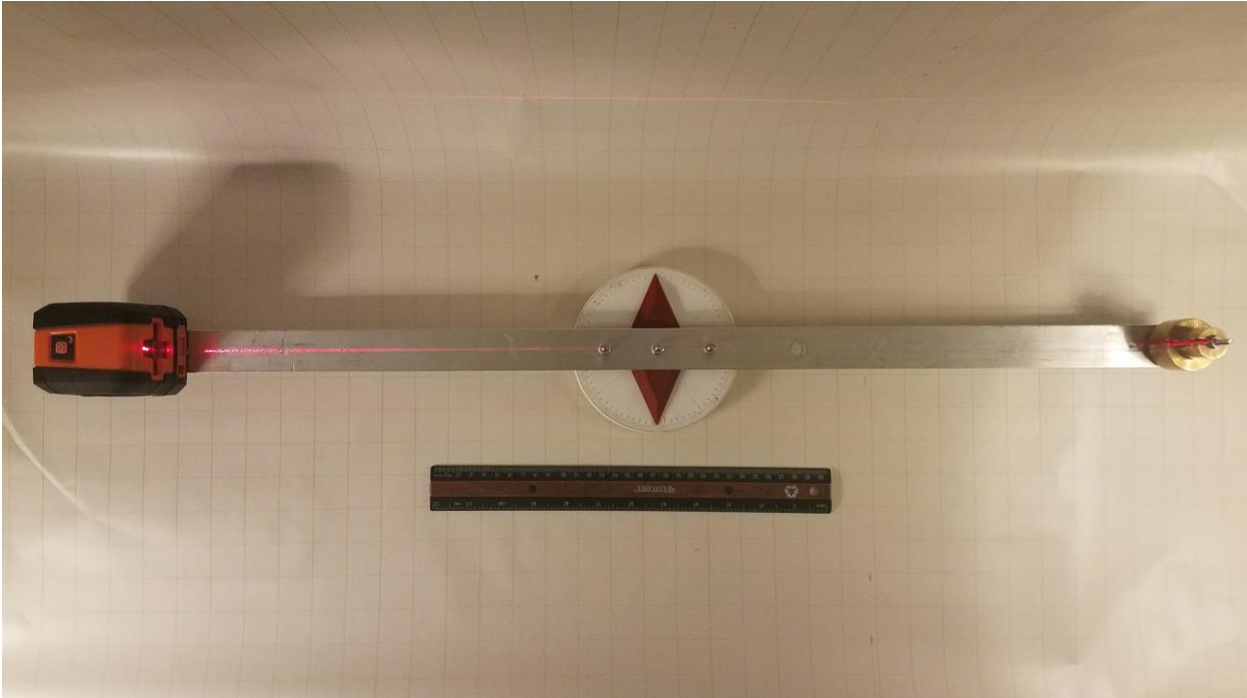
Appendix A: Detail of small-scale masses
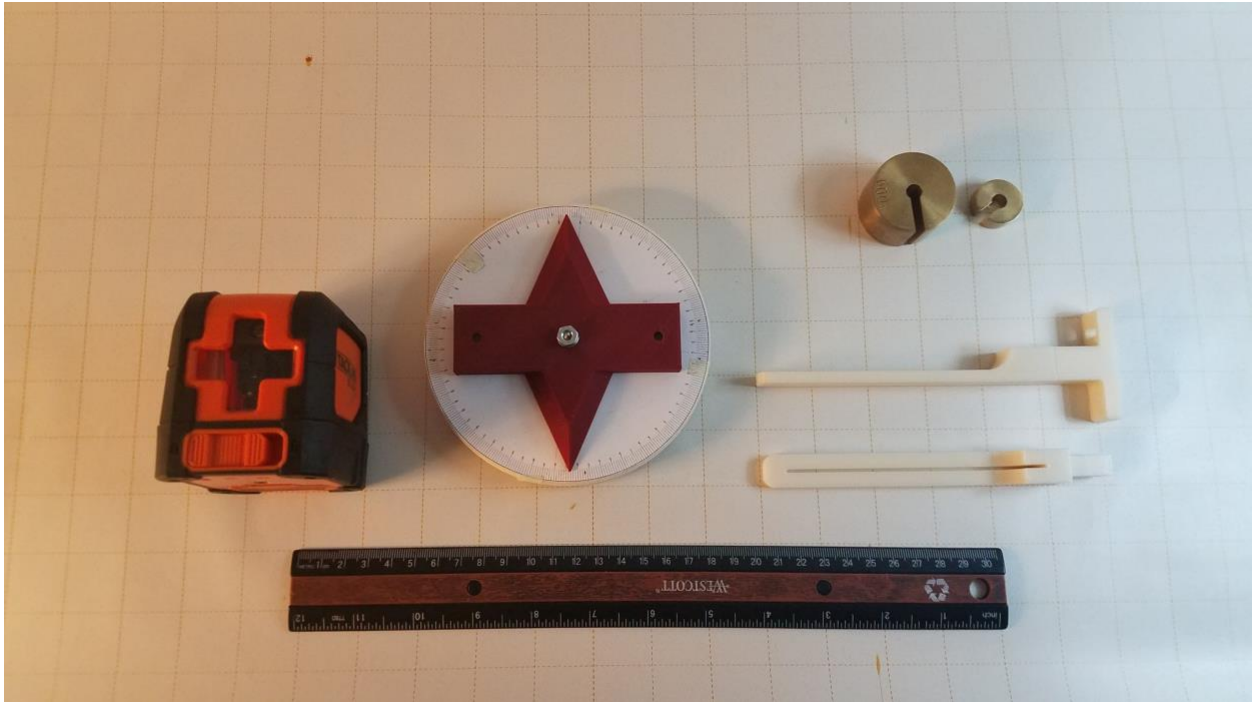

Appendix B: C-Clamp
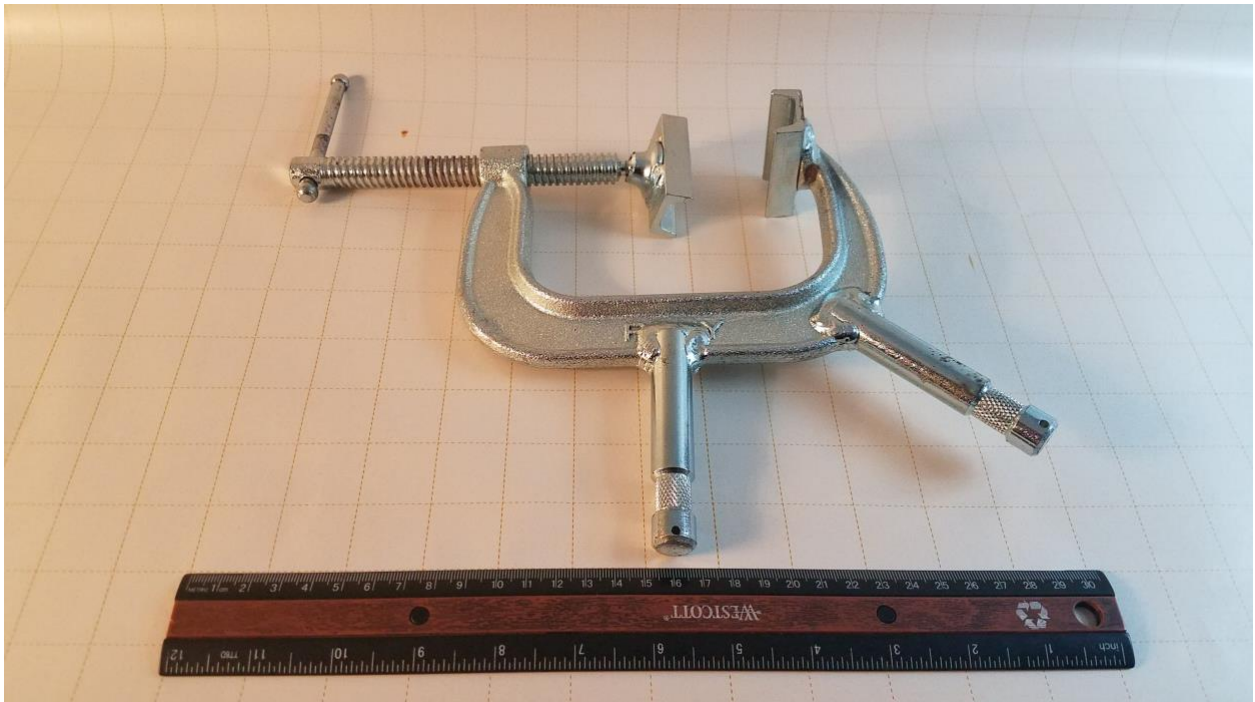
Appendix C: Suspension Block

Appendix D: Final PPC Assembly

Appendix E: PPC Components Detail

Appendix F: Utilized knot, recommended by the line manufacturer and printed on the package.