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A Study of Warping of Non-circular Shafts in Torsion

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A STUDY OF WARPING OF NON-CIRCULAR SHAFTS IN TORSION

ABSTRACT

This project is geared towards the study of warping as that takes place in non-circular shafts under torsion loading. For this purpose specimens of square cross-sections were used. .This activity constitutes a laboratory unit in Mechanics of Materials course taught at the junior level. The students are first introduced to the fact that the torsional stresses and deformations are the simplest for the circular cross-section, where the plane sections remain plane before and after the application of torque. The setup used for the experiment on torsion of a circular shaft was employed to study torsion of a shaft of square cross-section. The results for the circular cross-section were used to compute the shear modulus which was then used in the expressions for torsional stiffness of a square shaft by addressing warping. Warping in square shafts was demonstrated using geometrically similar Styrofoam specimens. Both the circular and square cross sections were further studied using the membrane analogy in which rubber membranes were stretched over square and round holes on plates pressurized by air from beneath the holes. The membranes were bulged forming "torsion" hills and the slopes at different locations around the contour were noted. These slopes were proportional to the shear stresses at the outer edge of the cross section.

INTRODUCTION

One of the topics in Mechanics of Materials is that of torsion of prismatic sections. The analysis of prismatic bars subjected to axial loads is available for arbitrary cross-sections. In contrast, simple analytical solutions for the deformation and stresses in a bar subject to axial torsion exist only for bars with circular cross-sections. The determination of stresses and deflections for the torsion of non-circular shaft involves equations that are quite complicated. However it is sometimes necessary to design shafts of non-circular cross-sections. As an example, the design of a connecting rod would require determination of torsional stresses in non-circular sections (typically I-sections) Such situations also exist in the design of various machine parts such as brackets and supports, which are not really shafts but are sometimes loaded in torsion. The analysis of torsion of shafts of non-circular sections is fairly complex because the assumptions that are valid for circular cross-sections do not apply here. For non-circular shafts under torsion, the plane cross-sections perpendicular to the shaft axis do not remain plane after twisting, and deformation takes place in the axial direction which is called warping. It is not the purpose of this paper to go through the analytical details associated with warping of non-circular shafts under torsion, but instead to develop an appreciation for this important effect through simple experiments.

This activity constitutes an extension of torsion test, typically performed for a torsion specimen of circular cross-section, by adding a test for torsion of a square cross-section. The purpose of the test for the torsion of a square cross-section was to demonstrate warping that is unique to sections that are non-circular cross-section.

WARPING

Warping in square cross-sections was clearly demonstrated by using Styrofoam specimens of square cross-section in which square grids were drawn along the faces of the specimen. Upon twisting the specimens take the shape as shown in Figure 1.

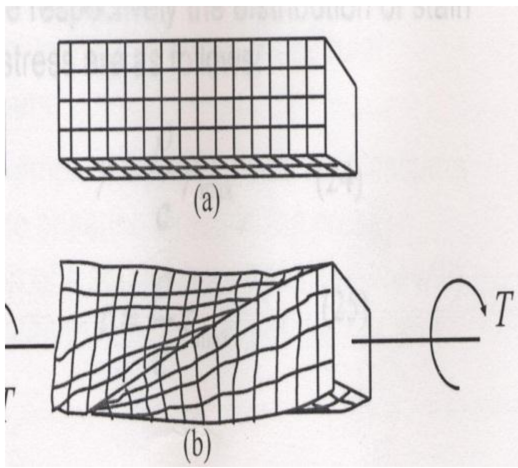

Figure 1 Demonstration of warping in Styrofoam specimens in torsion

Warping was further demonstrated by using the membrane analogy. Steel plates with a square hole as well as a round hole were used. Rubber sheets were rigidly clamped at the edges of the holes and made to bulge by applying pressure from beneath the plate. The resulting bulges (torsional hills) for the circular and the square holes are shown in Figures 2 and 3 respectively.

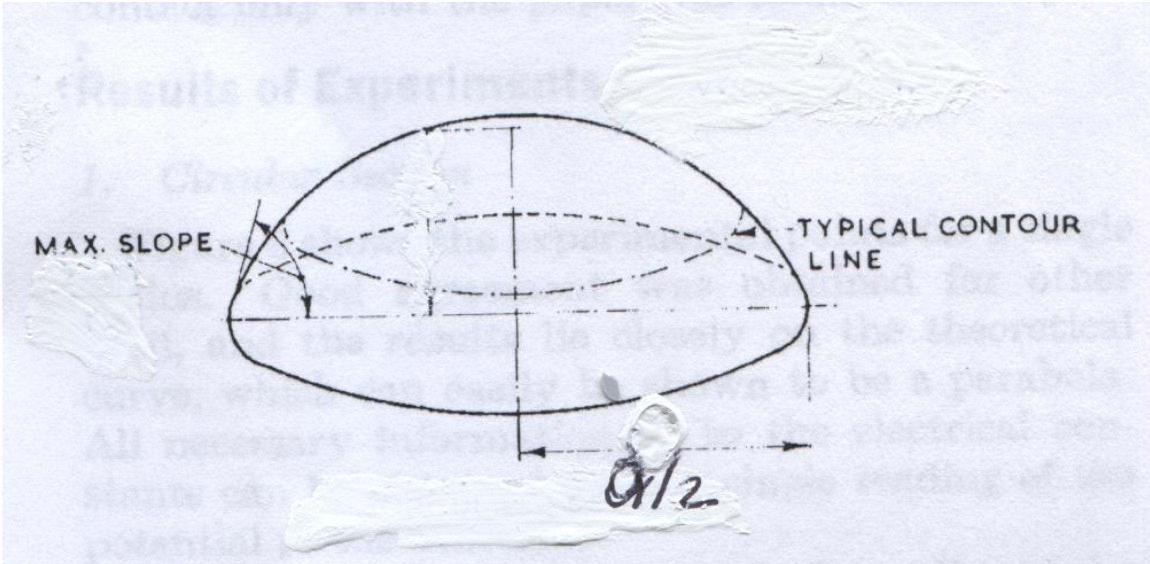

Figure 2 Torsion Hill for Circular Cross Section

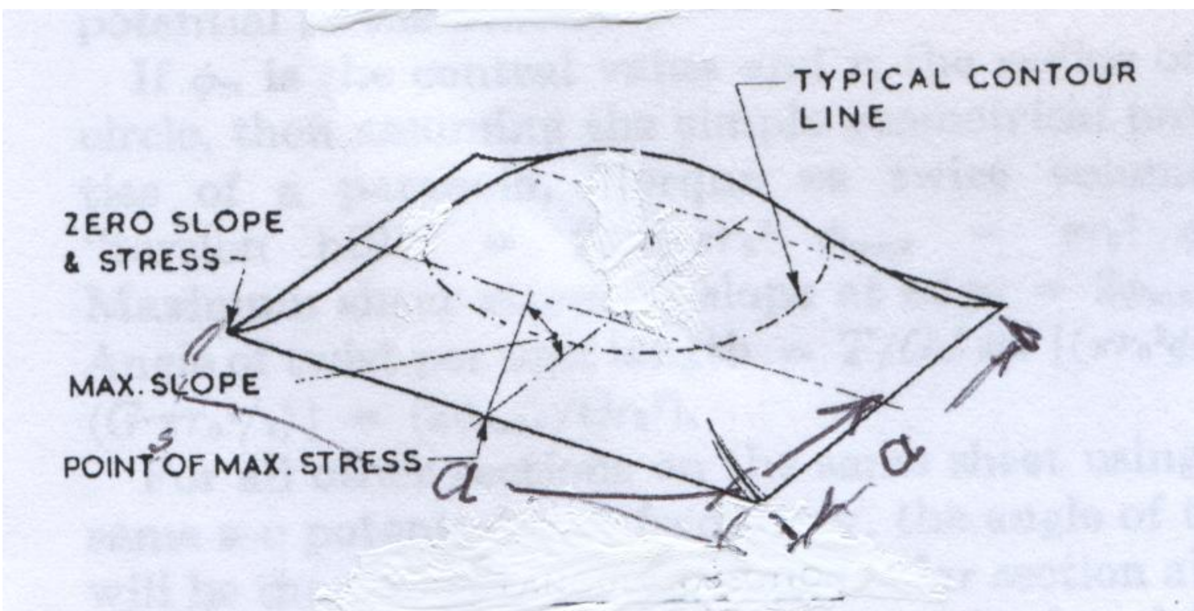

Figure 3 Torsion Hill for Square Cross Section

However for the square hole the membrane will be forced to lie flat (zero slope) at the corners and steepest slopes at the midpoints of the outer edges.

The students are made aware of the fact that the torsional shear stresses in a shaft of arbitrary cross section are proportional to the slopes of a suitably inflated flexible membrane (membrane analogy). Thus the stress is a maximum at the midpoint of the outer edges of a square shaft, and not at the corners. In the corners the membrane has a zero slope indicating a zero stress.

The students were also made aware of other tests using electrical analogy as reported in References [1] and [2].

<u>TORSION EXPERIMENTS</u>

Tinius Olsen torsion tester along with a reaction torque sensor were used.to perform torsion tests on 6061extruded aluminum bars of circular (0.25 inch in diameter) as well as square section (0.25 inch to a side). The best fit lines were drawn for the torque-twist curves for the square and round torsion specimens and are shown in Figure 5,

The torque-twist diagram for the round (circular) specimen was used to calculate the shear modulus of the material. This value was subsequently used to verify the analytical value for the torque-twist relationship for the square section.

The torsional stiffnesses (k) of the round and square specimens for a typical case were obtained experimentally (from the slopes of the lines in Figure 4) as 84 lb-in/rad and 118 lb-in/rad respectively.

Figure 4: Experimental Setup

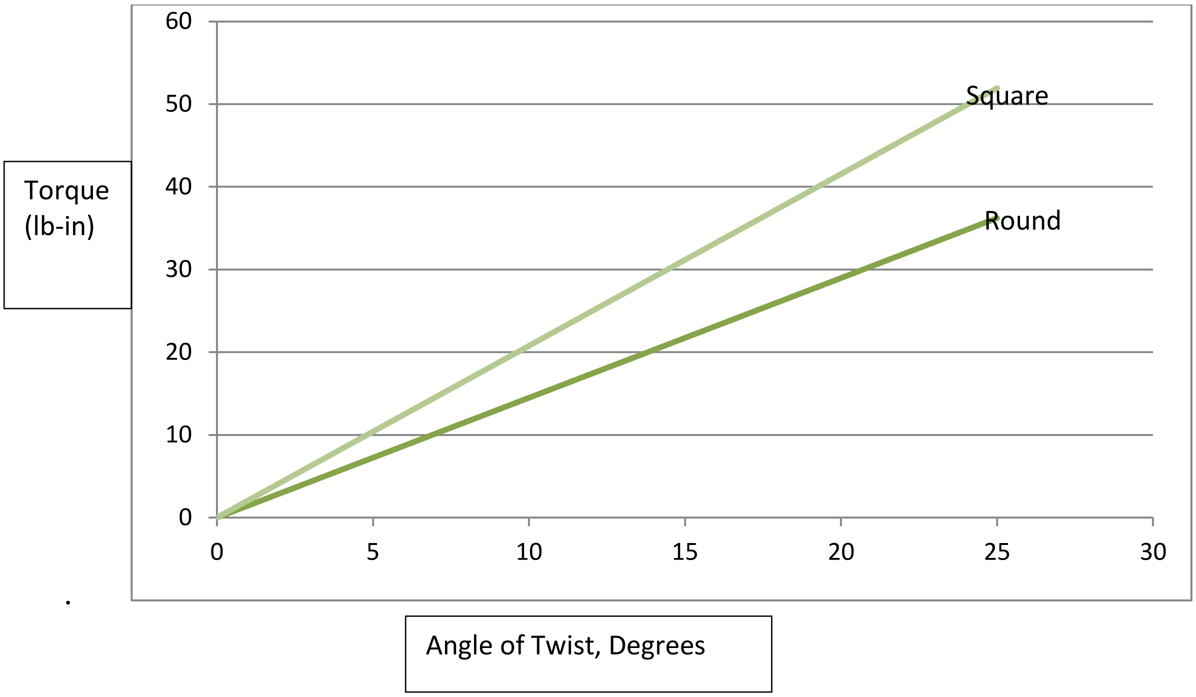

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Figure 5 Torque versus Angle of Twist for Round and Square Torsion Specimen

For the round specimen, from any text on Mechanics of Materials, such as, Reference [3]

$$k = \frac{GJ}{L} \qquad or\ 84 = \frac{G * (\pi * 0.25^4)}{32 * 18} \qquad (1)$$

Which gives G = 3.94 x 10^6 psi

For the square cross section, the determination of torsional stiffness requires consideration of warping which is available only in advanced texts on Mechanics of Materials, such as, Reference [4].

Stiffness for the square specimen, $k = \frac{G*(\propto * a^4)}{L} \quad or \quad 118 = \frac{3.94\ x\ 10^6 * (\propto * 0.25^4)}{18}$ (2)

This gives α = 0.138 which compares favorably with the analytical value of 0.1406 [4]

<u>CONCLUSIONS</u>

Warping has been demonstrated using (a) twisting Styrofoam specimens, (b) Membrane Analogy, and (c) Torsion experiments involving shafts of circular and square sections. Membrane analogy provides a mental picture of the state of stress. The membrane for a square shaft has been shown to have the greatest slope at the midpoint of the edges and not at the corners as is sometimes supposed. In the corner the membrane has a zero slope along both edges indicating zero stress. Experimental investigation on the torsion of square shaft provides a basis of comparison of its torsional stiffness with the analytical values that take warping into consideration. Future efforts in this area would be to measure strains using strain gages to compare with the analytical expressions for shear stresses, and also developing electrical analogy experiments similar to what has been reported in References [1] and [2].

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